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MEASUREMENT OF SMALL D.C. POTENTIALS AND CURRENTS IN HIGH RESISTANCE CIRCUITS BY USING VACUUM TUBES.

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ABSTRACT.

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Part 1. The grid current and plate current characteristics of three element vacuum tubes are described and the simple equations of the D.C. amplifier are developed with special attention given to the circuit for measuring photo-electric currents. The grid current characteristic is shown to play a very important part in the equations for sensitivity.

Part 2. The second part deals with three common output circuits, namely, (1) the single tube circuit with an auxiliary battery to balance out the normal plate current; (2) the single tube bridge circuit and (3) the two tube bridge circuit. Two "universal" shunts are described for controlling the galvanometer current. One of these is a "special shunt" which controls the sensitivity accurately and maintains the damping constant while the other is the well known Ayrton shunt. The disadvantages of using this shunt are discussed.

Part 3. Complete circuits for the measurement of potentials and currents by direct deflection and null methods are discussed and some of the advantages and limitations of each are pointed out.

Part 4. Under this heading a few suggestions are given as to points of technical procedure.

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INTRODUCTION.

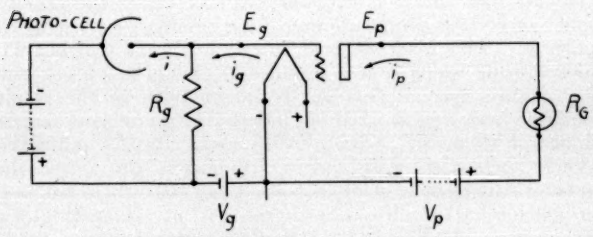
STUDIES requiring the measurement of small photoelectric currents have made it necessary to investigate the applicability of certain simple vacuum tube circuits to assist in the purpose. This work has been in progress some four years or more during which time a number of papers describing similar circuits have appeared. Although most of the results reported here were worked out independently of those papers, many of them are identical with those already published. Where such is the case and it is known to the writer, due acknowledgement is gladly accorded, but since there has been no exhaustive search of the literature, full acknowledgment to everyone who has reported similar results cannot be made. It is thought that there is enough new material contained in this paper to warrant its publication in spite of the fact that many of the results have already been published.

PART 1. GENERAL CONSIDERATIONS.

Of the many different types of circuits developed for the detection of small direct currents in high resistance circuits, only certain simple ones have been investigated. These depend on the observation of a change produced in the plate circuit of a three or four element vacuum tube as a result of a change in the grid potential. This change in grid potential is usually brought about by the "IR drop" produced in the grid resistance by the current being measured. When the circuit is used for potential measurements in high resistance circuits the change in grid potential is brought about directly. The simple circuit of Fig. 1 contains the essential elements. The operation of this circuit can be predicted at once from the "grid current *vs.* grid potential" and "plate current *vs.* grid potential" characteristics of the tube *taken with the operating resistance in the plate circuit*. Although theoretically it is not necessary to impose this requirement as regards the

plate circuit resistance, a great simplification is brought about by it in that it is then necessary to consider only the two curves mentioned above. If this condition is *not* imposed it is necessary to work with the family of curves showing the "grid current *vs.* grid potential" relationship for a number of different plate potentials and similarly for the "plate current *vs.* grid potential" relationship.

FIG. 1.

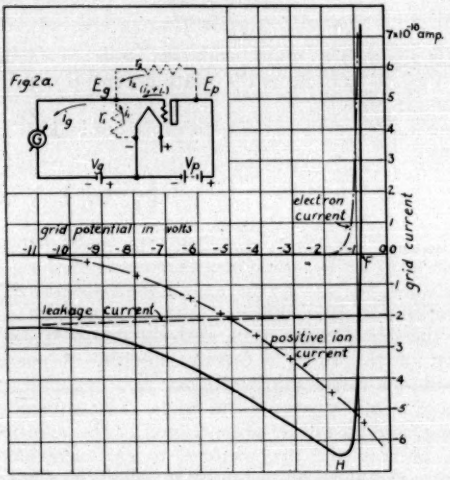


A. GRID CURRENT CHARACTERISTICS.

The grid circuit current characteristic shown by the solid line in Fig. 2 is a composite of at least four different currents. This current can be thought of as measured by means of the circuit shown in the schematic drawing Fig. 2*a*, inserted in Fig. 2.[1] The current flowing through the galvanometer for any given value of grid battery V_g and plate battery V_p is made up of the components described below. When a galvanometer of a low resistance is used the grid potential E_g will be practically equal to the "C" battery potential V_g. Since E_g will not be the same as V_g when a high resistance is introduced into the circuit, it will be well to recognize this distinction right at the beginning of this discussion. In the plate circuit there is, of course, the same sort of distinction between the "B" battery potential V_p and the plate potential E_p when the resistance in the plate circuit is not small compared with the plate impedance.

[1] An indirect method of determining the grid current which is essentially the same as that proposed by M. von Ardenne (*Jahrb. d. Drahtl. Telegr.*, *29*, 88, 1927) will be described later.

FIG. 2.



Grid current characteristics on UX-201-A vacuum tube.

a. Components of Current in Grid Circuit.

a1. Grid to filament leakage current.

The *leakage current* from the grid lead to the filament connections, socket support, etc., is

$$i_1 = -\frac{E_g}{r_1}, \qquad (1)$$

where r_1 is the grid to filament insulation resistance shown symbolically in the figure and E_g is the potential of the grid when referred to the negative end of the filament as zero. The negative sign used in this equation is necessary in order that the equation be consistent with the direction of flow of the current as indicated on the diagram of Fig. 2*a*.

a2. Plate to Grid Leakage Current.

The leakage current which flows through the galvanometer due to the combined batteries V_g and V_p of Fig. 2*a* is

$$i_2 = \frac{E_p - E_g}{r_2}, \tag{2}$$

where r_2 is the grid to plate insulation resistance symbolized by r_2 in Fig. 2*a* and E_p is the plate potential which in this case is the same as V_p.

The combined effect of these two types of leakage currents is given by

$$i_1 + i_2 = i_{12} = \frac{E_p}{r_2} - E_g\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \tag{3}$$

This leakage current is shown by the dashed line in the figure.[2]

*a*3. *Positive Ion Current.*

Since the gas can never be entirely removed during the evacuation of the tube, a certain amount of positive ion current will flow to the grid when it is at a negative potential with respect to other parts of the tube. The intensity of this positive ion current will be proportional to the number of gas molecules present and proportional to the electron current flowing from the filament to the plate, when the plate voltage is considerably higher than the ionization potential of the gas. Since the plate current depends on the grid potential, the positive ion current to the grid will depend on the grid potential.[3] This positive ion current i_+ is not a simple function of the grid potential and, therefore, can be represented only in a general way by

$$i_+ = f(E_g, E_p). \tag{4}$$

Graphically this relationship is shown by the curve of Fig. 2 designated "positive ion current."

[2] If the vacuum tube is constructed with a separate grid connection as is the case with a UX 222, a few turns of copper wire wound around the tube just above the bakelite base will serve as a guard ring. The potential of this ring can be maintained at the same value as the normal operating value of the grid potential and thus reduce the effective leakage current to a very small value. This leakage in tubes with a structure similar to the UX 201-A can be reduced by cutting out the base and covering the surface of the glass with wax or sulphur.

[3] Mulder and Razek, *J. O. S. A. and R. S. I.*, **18**: 466 (1929).

a4. Electron Current to Grid.

When the grid is only slightly negative with respect to the negative end of the filament, a certain number of electrons are able to overcome the adverse negative potential between the filament and the grid because of the high initial kinetic energy of these electrons due to the high temperature of the filament. The way in which this electron current depends on the grid potential can be represented approximately by the empirical equation [4]

$$i_- = i_o \epsilon^{aE_g}, \tag{5}$$

where E_g is the value of the grid potential which must be negative with respect to the negative end of the filament for the equation to hold. a and i_o are constants depending on the temperature of the filament and possibly on the plate potential.

The graphical representation given in Fig. 2 shows the form of the function which is the thing of primary interest.

The composite of these four currents gives the "grid circuit" current

$$i_1 + i_2 + i_+ + i_- = i_g, \tag{6}$$

This is shown graphically by the solid line of Fig. 2. This question of the grid current has been gone into at some length because it plays a very important part in a number of ways in the D.C. amplifier.

b. Definition of Grid Impedance.

In the practical problems of adapting this type of D.C. amplifier to definite purposes, it is not always necessary to measure the grid current characteristic but it is *very necessary* to visualize the "*form*" of the curve in order to understand what is really going on in the input circuit. In particular, it is important to know the slope of the curve at various grid potentials, i.e., the change in grid current for a small change in grid potential.

In the course of the discussion to follow, it will be necessary to use a quantity which is equal to the reciprocal of the slope of the grid current curve at some particular point. It has

[4] M. von Ardenne, *Jahrb. d. Drahtl. Telegr.*, **29**: 88 (1927).

become the accepted practice to designate the corresponding quantity in the plate circuit as the "plate impedance" of the tube. Let us therefore use the term "grid impedance" (Z_g), and let it be related to the slope of the grid current curve at any particular point by the equation

$$\frac{1}{Z_g} = \text{slope of the grid current curve} = \frac{\partial i_g}{\partial E_g}. \qquad (7)$$

From the curve we see that this "grid impedance" is positive for small negative values of E_g and is negative for larger negative values of E_g.

c. Definition of "Floating Potential."

The "floating potential,"[5] indicated by "F" in Fig. 2, is defined as the potential at which the grid current in the external circuit is *zero*. The grid impedance at this point has a definite positive value as given by the reciprocal of the slope of the tangent to the curve drawn through this point.

B. PLATE CURRENT CHARACTERISTICS.

The plate current characteristics of vacuum tubes are so well known and discussed in so many places[6] that it should not be necessary to do more than define a few of the common terms.

a. Mutual Conductance Defined.

In a vacuum tube, the mutual conductance (G_m) in the neighborhood of a given grid potential, is the slope of the "plate current *vs.* grid potential" curve taken with a *constant plate potential*. This definition expressed in equation form is

$$G_m = \left(\frac{\partial i_p}{\partial E_g}\right)_{E_p = \text{const.}}. \qquad (8)$$

[5] Sometimes denoted as the "potential of the free grid."

[6] Van der Bijl, "Thermionic Vacuum Tube," McGraw-Hill Book Co. Morecroft, "Principles of Radio Communication," John Wiley & Sons. The best source for practical information concerning vacuum tubes is the "Cunningham Tube Data Book" published by the E. T. Cunningham, Inc., 182 Second St., San Francisco.

FIG. 3.

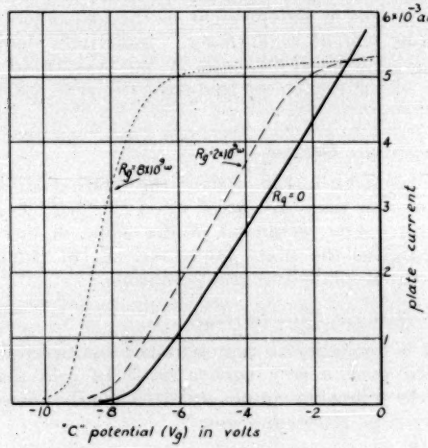
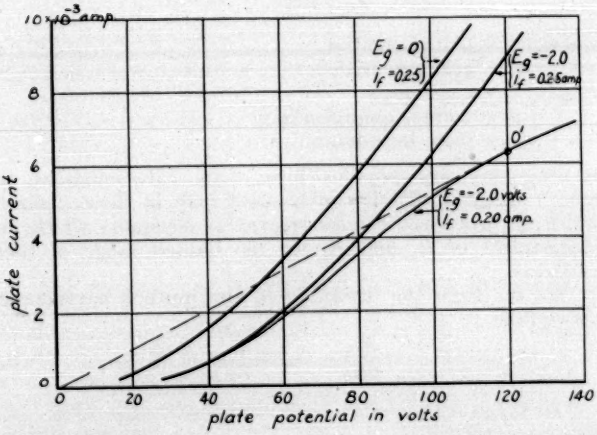

6×10⁻³ amp.
5
4
3
2
1
plate current
$R_g = 8 \times 10^9 \Omega$
$R_g = 2 \times 10^9 \Omega$
$R_g = 0$
-10
-8
-6
-4
-2
0
"C" potential (V_g) in volts

FIG. 4.


10×10⁻³ amp.
8
6
4
2
0
plate current
$E_g = 0$
$i_f = 0.25$
$E_g = -2.0$
$i_f = 0.25$ amp.
O'
$E_g = -2.0$ volts
$i_f = 0.20$ amp.
0
20
40
60
80
100
120
140
plate potential in volts

Mutual conductance so defined is dependent only on the tube characteristics and is independent of the associated circuits. The "*effective mutual conductance*," sometimes denoted by G_m', *depends* on the resistances in the associated circuits and can be either greater or less than the true mutual conductance G_m.

b. Plate Impedance Defined.

The curves given in Fig. 4 show the relationship between plate current and plate potential when the grid potential is held constant. The reciprocal of the slope of any one of these curves gives the plate impedance Z_p of the tube at the corresponding plate and grid potentials.

c. Tube Equation for Plate Current.

When it is necessary to represent the characteristics of a vacuum tube over a *very narrow range* of grid and plate potentials, the following simple equation nearly always gives a sufficiently close representation:[7]

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{9}$$

where E_p = plate potential,
E_g = grid potential,
E_o = constant,
μ = voltage amplification,
Z_p = plate impedance.

In using this very simplified form of the tube equation, we must remember that it can be used only in those cases in which E_o, μ and Z_p can be treated as constants which are independent of E_p and E_g for the limited range of their variation.

We see from the definition of the mutual conductance above that

$$\frac{\partial i_p}{\partial E_g} = G_m = \frac{\mu}{Z_p}. \tag{10}$$

[7] See Van der Bijl, page 50, for a more complete discussion of the limitations of this type of equation.

C. Operation of Simple D.C. Amplifier.

a. *Circuit Equations.*

We are now in a position to discuss the operation of the simple amplifier circuit of Fig. 1.

Since we are interested in the sensitivity of the amplifier we wish to know how the plate current of the tube depends on the grid potential. To compute the sensitivity, we use equation (9) and the Kirchoff Law equation for the plate circuit, which is

$$E_p = V_p - R_p i_p, \tag{11}$$

where V_p = "B" battery potential

and R_p = resistance in plate circuit including that of the battery, the galvanometer and any other resistances in the circuit.

Eliminating E_p we get

$$i_p = \frac{\frac{1}{Z_p}(V_p + \mu E_g + E_o)}{1 + \frac{R_p}{Z_p}}. \tag{12}$$

b. *Sensitivity to Voltage Determined.*

If we define the sensitivity S as the change in plate current due to a small change in grid potential we have

$$S \equiv \frac{\partial i_p}{\partial E_g} = \frac{\frac{\mu}{Z_p}}{1 + \frac{R_p}{Z_p}} = \frac{G_m}{1 + \frac{R_p}{Z_p}}. \tag{13}$$

The legitimacy of carrying out this variation, treating μ, Z_p and E_o as constants, is determined by the fact that the final equations and the experiments check for small variations of E_g. When R_p is very small compared with Z_p the "sensitivity" is equal to the mutual conductance G_m but if R_p is not small then the sensitivity is less than G_m because Z_p is *always* positive.

c. Sensitivity to Current Determined.

In many applications we are not interested directly in the change in plate current with a change in grid potential but in the change in plate current with a change in some current such as the photoelectric current.

From the Kirchoff Law equation for the grid circuit of Fig. 1, we have the equation

$$E_g = V_g - R_g(i_g - i) \tag{14}$$

where V_g = "C" battery potential,

R_g = external "grid leak" resistance,

i = photoelectric current.

We can perform a differentiation with respect to i in equation (14) and get

$$\frac{\partial E_g}{\partial i} = R_g - R_g \frac{\partial i_g}{\partial E_g} \times \frac{\partial E_g}{\partial i}. \tag{15}$$

Using the definition of Z_g given in equation (7) and rearranging we find

$$\frac{\partial E_g}{\partial i} = \frac{R_g}{1 + \frac{R_g}{Z_g}}. \tag{16}$$

This equation can also be written as

$$\frac{\partial E_g}{\partial i} = \frac{1}{\frac{1}{R_g} + \frac{1}{Z_g}}. \tag{17}$$

If R_g is small compared with Z_g then

$$\frac{\partial E_g}{\partial i} = R_g. \tag{18}$$

That is, the change in grid potential is equal to the change in "IR" drop over the grid resistance. On the other hand Z_g may be *positive* or *negative* depending on which part of the grid current characteristic we are operating. Then

the change in grid potential can be either *greater* or *less* than the "IR" drop change usually expected. If Z_g is negative and R_g is adjusted to make it approach the value $|Z_g|$ then $\frac{\partial E_g}{\partial i}$ will become very great because the denominator approaches zero.

Combining (13) and (16) we get

$$\frac{\partial i_p}{\partial i} = \frac{G_m}{1 + \frac{R_p}{Z_p}} \times \frac{R_g}{1 + \frac{R_g}{Z_g}}. \tag{19}$$

This equation gives the sensitivity as the *current amplification* of the one tube amplifier in terms of constants of the circuit. We can see from it exactly how each controllable factor enters to determine the amplifier sensitivity. For example, we see that when Z_g is *positive* and all other quantities constant except R_g, the sensitivity increases as R_g is increased but reaches an upper limit when R_g is made infinite, which is

$$\left(\frac{\partial i_p}{\partial i}\right)_{R_g=\infty} = \frac{G_m}{1 + \frac{R_p}{Z_p}} Z_g. \tag{20}$$

It is obvious that if R_g is made infinite the Z_g which must be taken for equation (20) is that found at the *floating potential*. It, therefore, follows that when two tubes are being compared the one with the higher "input impedance" at the floating potential will have the higher sensitivity if the mutual conductances are the same. The UX-222 (screen grid tube) has the highest input impedance of any tube we have measured in our laboratory. On the average, this has been found to be over 2000 megohms.

By taking advantage of the fact that the input impedance Z_g may be negative, a much higher sensitivity to photo-electric currents is possible. All that we have to do is to adjust the "C" battery potential so that the grid potential

$$E_g = V_g - i_g R_g \tag{21}$$

falls in the range in which Z_g is negative.

TABLE I.

Operating potential of grid.	E_g volts.	i_p amp.	G_m mho.	Z_p ohms.	R_g ohms assumed.	Z_g ohms.	R_g ohms arbitrary.	Current amplification computed by Eq. (19).
Floating.....	− 0.92	0	870×10^{-6}	8600	500	200×10^{6}	∞	166,000
Near point of inflection of grid current curve.....	− 3.0	5.5×10^{-10}	835×10^{-6}	9000	500	-16×10^{9}	200×10^{6}	161,000
	− 3.0	5.5×10^{-10}	835×10^{-6}	9000	500	-16×10^{9}	8000×10^{6}	12,700,000

A numerical example will serve to illustrate some of the conditions to be satisfied and the advantages to be had by this method of operation. If we use the grid current and plate current characteristics taken on the UX-201-A vacuum tube which served to give the data for Figs. 2, 3, and 4, we can calculate the current amplification for the single tube amplifier under certain conditions and thus illustrate the advantage of using the negative grid impedance. Table I lists the essential constants and the corresponding current amplification obtained by using equation (19).

We see from this computation that it is easily possible to obtain a current amplification which is 76 times the greatest to be had at the floating potential by operating at a grid potential for which the grid impedance is negative.

These data which were taken from the grid current characteristic of a particular UX-201-A vacuum tube can not be taken in any sense as representative of UX-201-A tubes in general. The amount of surface leakage and the amount of residual gas left in other tubes of the same type are likely to be very different because of changes in manufacturing methods from time to time. It is also possible that tubes produced by one manufacturer will differ in these respects from those produced by another. Obviously, it would be of considerable assistance to workers in this field if data were available by which some idea could be had as to the range of variation in the grid current characteristics which are likely to be met with in the common tubes, but such data are not available.

At this stage in the discussion it will perhaps be of interest to develop the equation which shows the rate of change in sensitivity with a change in grid impedance (Z_g), in order to show some of the difficulties resulting from an attempt to get extreme sensitivity by making $R_g = |Z_g|$ when Z_g is negative. Let us write equation (16) in the following form

$$s \equiv \frac{\partial E_g}{\partial i} = \frac{R_g}{1 + \frac{R_g}{Z_g}}. \qquad (22)$$

This equation gives s as the change in the grid potential E_g per unit change in photoelectric current. If we wish to know

the rate of change in s with a change in the tube input impedance Z_g, we can differentiate equation (22) and we get

$$\frac{\partial s}{\partial Z_g} = \frac{1}{\left(1 + \frac{Z_g}{R_g}\right)^2}. \qquad (23)$$

Consider first the case for which Z_g is *positive* and we operate in the region of the floating potential. Here we see that when R_g is infinite the right hand side of the equation is unity and the *change in sensitivity is directly proportional to the change in the input impedance.* If R_g is small compared with Z_g, then the right hand side of the equation is practically *zero* and the sensitivity is independent of Z_g.

In case Z_g is negative, then if R_g is small the sensitivity will be independent of Z_g. If $R_g = \frac{1}{2}|Z_g|$ the right hand member of equation (23) reduces to unity and it follows that the change in sensitivity is proportional to the change in input impedance. We can express the relation between the percentage change in s and the percentage change in Z_g by

$$\text{percent change in } s = \frac{(\text{percent change in } Z_g)}{1 + \frac{Z_g}{R_g}}. \qquad (24)$$

This equation shows that if a high sensitivity is obtained by making $\frac{Z_g}{R_g} = -1.1$ as would be the case if $Z_g = -220$ megohms and $R_g = 200$ megohms, the percentage change in sensitivity will be ten times as great as the percentage change in input impedance Z_g. Therefore, if we wish to keep the sensitivity constant to within 2 per cent., the input impedance for this particular case must be constant to within 0.2 per cent. Enough has now been said about grid current characteristics for the reader to take any particular grid current *vs.* grid potential curve and determine quite accurately the best operating conditions.

PART 2. SPECIAL OUTPUT CIRCUITS.

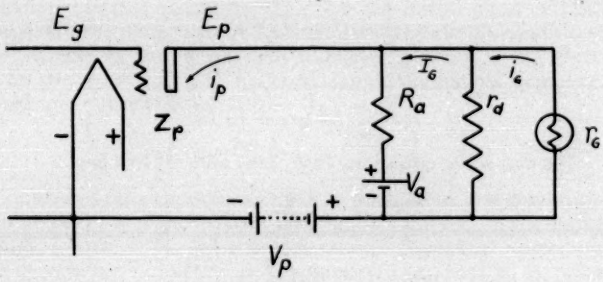
Practically all of the above discussion can be considered as the general circuit theory which forms the foundation for

all of the amplifiers under consideration. The part which follows will show the application of this general theory to a number of specific circuits. Some of the chief advantages and limitations of each circuit will be pointed out as we progress.

A. ONE TUBE CIRCUIT WITH AUXILIARY BATTERY TO SUPPRESS THE ZERO.

In applications in which a meter or a galvanometer is used in the plate circuit to detect the changes in plate current brought about by the changes in grid potential imposed by the outside circuits, it is usually advantageous to have the meter read "zero" when there is no current in the photoelectric circuit, for instance. This result can be brought

FIG. 5.



about mechanically or electrically. If the meter is connected directly in the plate circuit, the meter will read the normal plate current. It is obvious that by turning the supporting suspension or the restoring spring through a sufficient angle the meter will read "zero." This would be a mechanical method for "suppressing the zero." There are a number of ways of bringing about this suppression of zero electrically. One of the simplest ways is to use a small auxiliary battery and a control resistance across the meter or galvanometer as shown in Fig. 5.

a. Circuit Equations.

In order to obtain equation (12) which showed the plate current as a function of grid potential, in the example worked

out above it was necessary to have only two equations, (1) the tube equation and (2) the simple Kirchoff equation of the plate circuit. In the present case the procedure is exactly the same since we start again with the tube equation. In place of a single plate circuit equation we now need more than one because it is now necessary to consider more than one circuit "mesh." The following simple equations are all that are necessary.

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_a), \tag{25}$$

$$E_p = V_p - I_G R_G, \tag{26}$$

$$V_a = i_p R_a - I_G(R_a + R_G), \tag{27}$$

where

$$\frac{1}{R_G} = \frac{1}{r_d} + \frac{1}{r_G}, \tag{28}$$

$$i_G = I_G\left(\frac{r_d}{r_d + r_G}\right). \tag{29}$$

We can solve equations (25), (26) and (27) to get

$$I_G = \frac{\frac{1}{Z_p}(V_p + \mu E_g + E_a) - \frac{V_a}{R_a}}{1 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_a}\right)}. \tag{30}$$

b. Sensitivity to Grid Voltage Changes.

The sensitivity to voltage is therefore

$$\frac{\partial I_G}{\partial E_g} = \frac{G_m}{1 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_a}\right)}. \tag{31}$$

When we substitute equations (28) and (29) in (31) we get

$$\frac{\partial i_G}{\partial E_g} = \frac{G_m}{1 + r_G\left(\frac{1}{Z_p} + \frac{1}{R_a} + \frac{1}{r_d}\right)}. \tag{32}$$

In case we adjust R_a and r_d so that the galvanometer is critically damped and R_{cd} is the external critical damping resistance, then from Fig. 5 we see that

$$\frac{1}{R_{cd}} = \frac{1}{Z_p} + \frac{1}{R_a} + \frac{1}{r_d}, \tag{33}$$

assuming that the battery resistance can be neglected.

We conclude from equations (32) and (33) that the sensitivity when the galvanometer is critically damped depends only on the mutual conductance and the ratio of the galvanometer resistance to the critical damping resistance.

Referring to equation (27), we see that the value of R_a is determined by the initial plate current, call it i_{po}, and the auxiliary battery potential V_a. By the initial plate current we mean the plate current flowing under the circuit conditions for which we want the galvanometer current to be zero. The *minimum* value of V_a, consistent with critical damping, is given by the relationship

$$V_{a\,(\text{min.})} = i_{po}\left(\frac{R_{cd}Z_p}{Z_p - R_{cd}}\right). \tag{34}$$

Equation (34) states in analytical form, that the minimum value of V_a is equal to the "IR" drop over the lowest value of R_a which is consistent with critical damping. After V_a is determined consistent with equation (34), that is, V_a must be greater than $V_{a\,(\text{min.})}$, we have R_a given by equation (35).

$$R_a = \frac{V_a}{i_{po}}. \tag{35}$$

From equation (33) we find

$$\frac{1}{r_d} = \frac{1}{R_{cd}} - \frac{1}{Z_p} - \frac{1}{R_a}. \tag{36}$$

We have thus determined the constants of our circuit to give the maximum sensitivity to grid voltage consistent with critical damping using a tube of mutual conductance G_m, and a galvanometer of resistance r_g and external critical damping resistance R_{cd}. This sensitivity is

$$\frac{\partial i_G}{\partial E_g} = \frac{G_m}{1 + \frac{r_G}{R_{cd}}}. \qquad (37)$$

In order to illustrate the meaning of equation (37) let us consider a numerical example in which the galvanometer has a resistance of $r_G = 500$ ohms and a critical damping resistance $R_{cd} = 2000$ ohms and a vacuum tube with a mutual conductance $G_m = 900$ micro-mhos. Then

$$\frac{\partial i_G}{\partial E_g} = \frac{900 \times 10^{-6}}{1 + \frac{500}{2000}} = 0.72 \times 10^{-3} \text{ amp. per grid volt.}$$

A simple calculation shows at once that in case the resistance of the circuit in which the unknown E.M.F. originates is 2000 ohms, the current through the galvanometer when used with the vacuum tube circuit of Fig. 5, will be only 1.44 times as great as the current which would flow through the galvanometer if it were connected directly. If the galvanometer sensitivity is 330 megohms, then the overall sensitivity of the system measured in millimeters deflection, is 238 mm. per millivolt applied to the grid.

B. WHEATSTONE BRIDGE CIRCUIT WITH SINGLE TUBE.[8]

a. Circuit Equations.

Another method of arranging the circuit so that no current flows through the galvanometer for the "zero" condition but does flow when this condition is disturbed, comes directly from the simple Wheatstone bridge circuit. In Fig. 6 we have this circuit with resistances in three of the arms and a vacuum tube in the remaining one. It is easy to show that with this circuit the galvanometer balance or even the galvanometer deflection will be *independent* of the "B" battery potential (V_p) over a certain limited range. The solution of the following simple equations gives this result. Using the vacuum tube equation (9) and the Kirchoff equations for the Wheatstone bridge we can write:

[8] Other single tube circuits have been discussed by Razek and Mulder, *O. S. A. and R. S. I.*, **18**: 460 (1929).

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{9}$$

$$i_x = \frac{E_x}{R_x}, \tag{38}$$

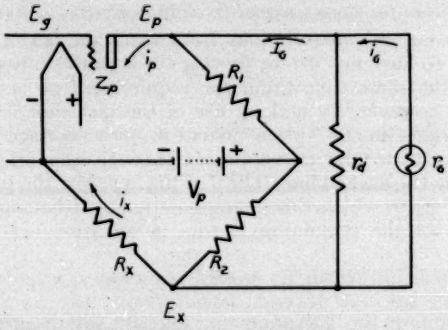
$$E_p = V_p - (i_p - I_G)R_1, \tag{39}$$

$$E_x = V_p - (i_x + I_G)R_2, \tag{40}$$

$$O = R_1(i_p - I_G) - I_G R_g - R_2(i_x + I_G). \tag{41}$$

The meaning of the symbols is given by the circuit schematic Fig. (6). The combined resistance of the galvanometer r_g and the parallel resistance r_d is R_G and is given by equation (29).

FIG. 6.



Solving for the current I_G we get:

$$I_G = \frac{(R_1R_x + R_1R_2)(E_o + \mu E_g) - (R_2Z_p - R_1R_x)V_p}{R_1Z_pR_x + R_2Z_pR_x + R_1R_2Z_p + R_1R_2R_x + R_G(Z_p + R_1)(R_x + R_2)}. \tag{42}$$

b. Plate Battery Compensation Conditions.

We see at once from equation (42) that I_G is independent of V_p if

$$R_2Z_p = R_1R_x. \tag{43}$$

In order that this be true the constants of the tube equation Z_p, μ and E_o must all be independent of E_p over the range corresponding to that over which V_p is assumed to vary. This condition can be met in most cases for changes in V_p of one to three per cent.

c. Voltage Sensitivity.

In order to compute the sensitivity of this amplifier let us take $R_1 = R_2$, then $R_x = Z_p$ and we have, remembering that I_G in this case is the current through the galvanometer and its damping resistance and that R_G is r_G in parallel with r_d,

$$\frac{\partial I_G}{\partial E_g} = \frac{G_m}{2 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}. \tag{44}$$

d. Condition for Zero Current in Galvanometer.

It is not at once obvious from equation (42) that it is possible to have no current flowing through the galvanometer and at the same time fulfill the requirement of (43). It is possible however, by making use of the fact that for a very limited range in the "plate current *vs.* plate voltage" curve,[9] the tangent to the curve passes through the origin as is shown by the dashed line "*OO'*" of Fig. 4. For the particular value of E_g to which this characteristic curve belongs, call it E_g', one of the remaining factors in equation (42) is zero. That is

$$E_o + \mu E_g' = 0 \tag{45}$$

and therefore the galvanometer current will be zero when $E_g = E_g'$ and yet we shall be able to meet the requirement set down by equation (43). When E_g changes from the initial value E_g' by an amount ΔE_g the total current (I_G) in the galvanometer circuit will be given by equation (44).

The sensitivity equation (44) is expressed in terms of the

[9] The bending over of the characteristic shown in Fig. 4 is due to the fact that the filament current has been reduced to bring about a limitation of current with increasing plate potential owing to "saturation." The circuit of Fig. 6 could be modified by placing a battery in series with R_x and thus make it possible to operate on a "straighter" portion of the "plate current vs. plate potential" curve.

current flowing through the galvanometer and the damping resistance in parallel. In order to find the sensitivity in terms of the current flowing through the galvanometer we must use equations (47) and (48) below. The current i_G through the galvanometer alone is of course

$$i_G = I_G\left(\frac{r_d}{r_d + r_G}\right), \tag{46}$$

r_d = damping resistance,

r_G = galvanometer resistance.

This equation solved for I_G is

$$I_G = i_G\left(\frac{r_d + r_G}{r_d}\right). \tag{47}$$

The parallel resistance of the galvanometer and shunt is obviously

$$R_G = \frac{r_d r_G}{r_d + r_G}. \tag{48}$$

When these equations are substituted into equation (44) we get

$$S_i = \frac{\partial i_G}{\partial E_g} = \frac{G_m}{2\left[1 + r_G\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]} \tag{49}$$

as the sensitivity equation for the circuit of Fig. 6 expressed in terms of the galvanometer current i_G. By inspection of Fig. 6 and remembering that $R_x = Z_p$ and $R_1 = R_2$ we see that the galvanometer is critically damped when

$$\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d} = \frac{1}{R_{cd}} \tag{50}$$

where R_{cd} is the external critical damping resistance. Making this substitution, we get

$$S_i = \frac{G_m}{2\left(1 + \frac{r_G}{R_{cd}}\right)}. \tag{51}$$

e. Sensitivity of Circuits 5 and 6 Compared.

If we compare the sensitivity of the circuit shown in Fig. 6 and given by equation (51) with that for the circuit of Fig. 5 given by equation (37), we get

$$\frac{S_6}{S_5} = \frac{\dfrac{G_m}{2\left(1 + \dfrac{r_G}{R_{cd}}\right)}}{\dfrac{G_m}{1 + \dfrac{r_G}{R_{cd}}}} = \frac{1}{2}. \tag{52}$$

In other words we have sacrificed exactly half of our sensitivity in order to introduce the bridge circuit which, however, is superior to the simple circuit in that the zero or the deflection is independent of the "B" battery over a certain limited range of variation while in the simple circuit of Fig. 5 a change in either the "B" battery potential (V_p) or the auxiliary battery (V_a) will produce a change in the galvanometer reading.

C. WHEATSTONE BRIDGE CIRCUIT USING TWO VACUUM TUBES.

a. Conditions for "B" Battery Compensation.

The circuit shown in Fig. 6 can be modified as shown in Fig. 7 by the introduction of a vacuum tube in place of the resistance arm R_x. It will be shown below that this change in the circuit leaves the form of the equation for the sensitivity unchanged and modifies the condition for "B" battery compensation only to the extent that Z_{px} replaces R_x in equation (43). This gives

$$R_2 Z_p = R_1 Z_{px} \tag{53}$$

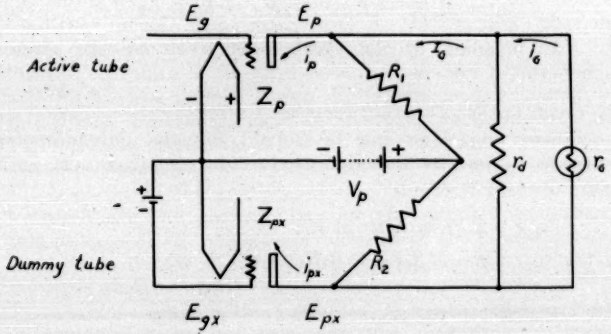
or

$$\frac{Z_p}{Z_{px}} = \frac{R_1}{R_2}. \tag{54}$$

Here Z_{px} is the plate impedance of the tube which has been introduced in the bridge in place of R_x. In the discussion

above it was assumed that $R_1 = R_2$ and R_x was adjusted to have the value $R_x = Z_p$. That order of procedure cannot be used in this case because the value of the plate impedance Z_{px} is not so easily controlled as the value of R_x was before. It, therefore, follows that we must meet the condition required by equation (54) by adjusting $\frac{R_1}{R_2}$. Fortunately it is not necessary to know Z_p and Z_{px} exactly. All that is necessary

FIG. 7.



is that we have R_1 or R_2 adjustable and have a means of altering the "B" battery potential (V_p) through the narrow range over which we are required to get the best possible compensation.[10]

b. Circuit Equations.

In order to develop the equations which give support to the statements just made as regards independence of "B" battery (V_p), sensitivity, etc., we must start out with five equations just as we did in the one tube bridge circuit discussed above. These equations were (9), (38), (39), (40)

[10] This method must be used with a certain amount of care because it is possible to get a compensation at the end points and not have compensation for intermediate values of (V_p) owing to the curvature of the characteristic.

and (41) and the corresponding set here are the following:

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{55}$$

$$i_{px} = \frac{1}{Z_{px}}(E_{px} + \mu_x E_{gx} + E_{ox}), \tag{56}$$

$$E_p = V_p - (i_p - I_G)R_1, \tag{57}$$

$$E_{px} = V_p - (i_{px} + I_G)R_2, \tag{58}$$

$$O = R_1(i_p - I_G) - I_G R_G - R_2(i_{px} + I_G). \tag{59}$$

The meaning of the symbols is given by the circuit schematic of Fig. 7.

c. Condition for Zero Galvanometer Current.

These equations can be solved for the galvanometer current I_G just as was done before in equation (42) with the following result

$$I_G = \frac{\begin{array}{r}(R_1Z_{px} + R_1R_2)(\mu E_g + E_o) \\ - (R_2Z_p + R_1R_2)(\mu_x E_{gx} + E_{ox}) \\ - (R_2Z_p - R_1Z_{px})V_p\end{array}}{\begin{array}{r}R_1Z_pZ_{px} + R_2Z_pZ_{px} + R_1R_2Z_p + R_1R_2Z_{px} \\ + R_G(Z_p + R_1)(Z_{px} + R_2)\end{array}} \tag{60}$$

and the sensitivity is

$$\left(\frac{\partial I_G}{\partial E_g}\right)_7 = \frac{G_m}{2 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}, \tag{61}$$

when $R_1 = R_2$ and $Z_p = Z_{px}$. This is the same as was found above in equation (44). When equations (47) and (48) are used so as to express the sensitivity in terms of the galvanometer current i_G, this equation obviously reduces to the same as (49) which is

$$S_7 = \left(\frac{\partial i_G}{\partial E_g}\right)_7 = \frac{G_m}{2\left[1 + r_G\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]}. \tag{62}$$

Again if the galvanometer is critically damped and R_{cd} is the external critical damping resistance we have

$$S_7 = \frac{G_m}{2\left(1 + \frac{r_G}{R_{cd}}\right)}. \tag{63}$$

From equation (60) we see that the condition for operation independent of V_p is that given above in equation (53) or (54). Assuming this condition to be satisfied, we have the further condition which we wish to impose, that there be no current flowing through the galvanometer for some definite condition in the grid circuit such as, for example, that for which there is no photoelectric current. For the arbitrary "initial" condition, the grid potential of the "active" tube will have some definite value E_g' which is determined by the circuit constants such as the grid resistance R_g, the "C" battery potential V_g, etc. (see Fig. 1). The galvanometer current can be brought to zero by properly adjusting the potential of the grid E_{gx} of the "dummy" tube. Later we shall use the phrase "balance the circuit" to indicate this process of adjusting some element of the circuit such as the grid potential E_{gx} until the current through the galvanometer is zero.

When we assume the condition, given by equation (53), to be satisfied, we see from equation (60) that the galvanometer current will be zero when the sum of the remaining two terms of the numerator of that equation is zero. This we have expressed in the equation.

$$(R_1Z_{px} + R_1R_2)(\mu E_g' + E_o) - (R_2Z_p + R_1R_2)(\mu_x E_{gx} + E_{ox}) = 0. \tag{64}$$

Referring to equation (53), which we are assuming to be satisfied, we see by adding R_1R_2 to each side of the equation that

$$R_1Z_{px} + R_1R_2 = R_2Z_p + R_1R_2. \tag{65}$$

Therefore, if equations

$$\mu E_g' + E_o = \mu_x E_{gx} + E_{ox} \tag{66}$$

and

$$R_2Z_p = R_1Z_{px} \tag{53}$$

are satisfied, the galvanometer current is zero and the system is independent of "B" battery changes over the limited range for which the "constants" of the system are independent of "B" battery.

d. "Cut and Try Method" of Adjustment.

The fact that the current through the galvanometer is zero when equations (53) and (66) are satisfied does not mean that we need to know the constants of the tubes actually used in this two tube bridge circuit any more than it did above in the one tube bridge circuit. In the single tube circuit we saw that in order to accomplish a balance of the circuit, it was necessary to operate on the part of the "plate current *vs.* plate potential" curve of Fig. 4 at O'. At this part of the characteristic the plate impedance Z_p changes rather rapidly with a change in E_p and therefore limits the range over which satisfactory "B" battery compensation can be obtained. When we use the two tube circuit we can operate over the "straight" portion of the characteristic where the change in Z_p and E_p is very small and still meet the requirements of equations (53) and (66) simultaneously. This is accomplished by adjusting the resistance ratio $\frac{R_1}{R_2}$ and the value of the grid potential on the "dummy" tube until both conditions are met. In practice this adjustment is of the "cut and try" type. That is, we arbitrarily set $R_1 = R_2$ and adjust E_{gx} to balance the circuit giving no current through the galvanometer. Then we change the "B" battery potential (V_p) by 1.5 volts or more and observe the amount and direction of the galvanometer deflection.[11] We then change R_2 by 1000 ohms,[12] for instance, rebalance the circuit by altering E_{gx} and again test for stability. If the galvanometer swings less far but in the same direction we see that we have altered the resistance ratio $\frac{R_1}{R_2}$ in the right direction but not enough. By continuing this "cut and try" method it is nearly always possible to obtain

[11] See footnote 10.

[12] If we wish to keep the galvanometer damping more nearly the same for all settings, we should alter R_1 also, keeping $(R_1 + R_2)$ constant.

perfect stabilization for moderate changes in "B" battery potential. The fact that, with the two tube circuit, we can operate on the "*straight*" part of the curve makes it possible to realize perfect stability over a wider range of "B" battery variation than can be realized in the simple one tube bridge circuit.

e. Advantages of Two Tube Circuit.

There are other advantages to the two tube circuit which deserve to be mentioned. Two of these are (1) compensation for filament voltage changes and (2) approximate compensation for tube aging. In general, the plate current through a tube gradually decreases with time when the plate and grid potentials and the filament current are held constant. It is obvious that that part of the drift in the galvanometer current which is due to this aging, will be less in the two tube circuit since we can assume that the rate of aging in two similar tubes is approximately the same.

f. Conditions for Filament Battery Compensation.[13]

If we had included filament voltage E_f as one of the variables of the tube equation we would have had the following equations instead of (55) and (56) as above.

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o') + \alpha E_f, \tag{67}$$

$$i_{px} = \frac{1}{Z_{px}}(E_{px} + \mu_x E_{gx} + E_{ox}') + \alpha_x E_f. \tag{68}$$

The two new constants are defined by

$$\alpha \equiv \frac{\partial i_p}{\partial E_f}, \tag{69}$$

$$\alpha_x \equiv \frac{\partial i_{px}}{\partial E_f}. \tag{70}$$

[13] For an extended discussion of compensation methods including compensation for grid battery changes see:

C. E. Wynn-Williams, *Phil. Mag.*, **6**: 324 (1928).

J. Brentano, *Phil. Mag.*, **7**: 685 (1929).

J. M. Eglin, *J. O. S. A. and R. S. I.*, **18**: 393 (1929).

J. Razek and P. J. Mulder, *J. O. S. A. and R. S. I.*, **19**: 390 (1929).

Here again we must rely on experiment for the justification of the statement that α and α_x can be treated as constants which are independent of E_p, E_g and E_f over narrow ranges of the variables.

If we carried through the calculations to get the equation equivalent to (60), we would find the following term in the numerator along with those already discussed

$$E_f\{(R_1Z_{px} + R_1R_2)Z_p\alpha - (R_2Z_p + R_1R_2)Z_{px}\alpha_x\}. \quad (71)$$

If this term is zero then the system will be independent of filament voltage and all of the relations developed above will still hold perfectly well. Take the first, namely that required in order to have independence from "B" battery changes which was $R_1Z_{px} = R_2Z_p$ and it follows that

$$Z_p\alpha = Z_{px}\alpha_x \quad (72)$$

if the system is to be independent of filament voltage. In terms of the resistances we have

$$\frac{\alpha_x}{\alpha} = \frac{R_1}{R_2}. \quad (73)$$

By operating the two filaments in parallel but with a small variable resistance in series with one of them, the α of one of the tubes can be adjusted to meet condition (73). There are other ways of building the circuit so as to control one of the α's but the condition (73) must always be met. Again it is no more necessary to know the new constants of the tube equation than it was before. The balance is accomplished by the "cut and try" method just as before but this time we must give small arbitrary changes to the filament battery to test the compensation. In the laboratory we have found that it is possible to make the compensation so perfect that little or no deflection of the galvanometer could be measured with a filament change of as much as one or two per cent. while without the compensation a change of one-tenth of a percent caused a change in the galvanometer reading.

D. SUMMARY.

The part of this paper just completed deals with three important circuits used in single stage D.C. amplifiers. The

entire attention has been devoted to the conditions in the plate circuit or the filament circuit and little or nothing has been said about the grid circuit. Many of the general principles which determine the grid circuit connections have been recorded in the first part of this paper. The detailed application of these principles will be taken up in the following section. The one tube circuit with an auxiliary battery to give a "supressed zero" was shown to be the most sensitive one stage amplifier of the three. The bridge circuits which followed were shown to have a sensitivity one-half as great as the above. With the single tube bridge circuit, it was shown that under certain definite conditions small variations in "B" battery could be compensated for. The range for which this compensation is possible was shown to be very limited. The bridge circuit with two tubes was shown to have several advantages over the single tube bridge circuit with no additional loss in sensitivity. These advantages were (1) practically perfect "B" battery compensation over a rather wide range, (2) filament battery compensation over a fairly wide range, and (3) approximate compensation for aging of the tubes.

E. CONTROL OF GALVANOMETER DEFLECTION.

a. Special Universal Shunt.

Whenever a sensitive galvanometer is used in any of the three circuits described, it is usually necessary to have some means for controlling the galvanometer response in a way which keeps the damping correct. In ordinary high resistance circuits an Ayrton shunt is suitable for this purpose. Under certain conditions an Ayrton shunt can be used here also, but as will be shown later a certain amount of sensitivity must be sacrificed in order to do this. It is possible, however, to control the current through the galvanometer in either the bridge circuit or that with the auxiliary battery and keep the damping correct without this loss in sensitivity. The following computation shows how to determine the constants of the circuit. Consider a circuit like that shown in Fig. 7 with the galvanometer replaced by the network shown in Fig. 8. When the contact is at 1, resistance "a"

is zero and resistance "b" infinite. The sensitivity is

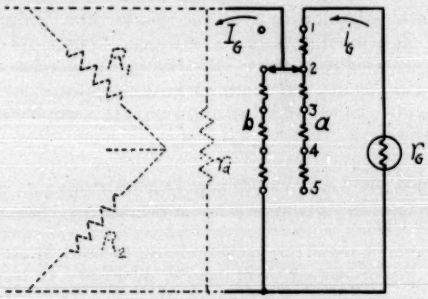
$$S_{\max} = \frac{\partial i_G}{\partial E_g} = \frac{G_m}{2\left[1 + r_G\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]}. \qquad (74)$$

This result comes from equation (62) and gives the maximum sensitivity. The damping is correct when

$$\frac{1}{R_{cd}} = \frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}, \qquad (75)$$

where R_{cd} is the external resistance for critical damping.

FIG. 8.



Circuit with special universal galvanometer shunt.

As we move the contact to successive positions, 2, 3, etc., resistances a and b have two functions to perform. They must be so proportioned that the damping will remain unchanged and the sensitivity reduced in definite amounts. If a and b are always adjusted so as to satisfy equation (76) the damping will remain constant.

$$R_{cd} = a + \frac{bR_{cd}}{b + R_{cd}}. \qquad (76)$$

If S_n is the sensitivity for some combination of a and b

consistent with (76), we can define the "sensitivity ratio" n as

$$n = \frac{S_{\max}}{S_n}. \tag{77}$$

If we let I_G be the current through the galvanometer and the shunt b together, then the current i_g through the galvanometer will be

$$i_G = I_G \frac{b}{a + b + r_G}. \tag{78}$$

Since the resistance of the galvanometer along with its series and shunt resistances is

$$R_G = \frac{b(a + r_G)}{a + b + r_G}, \tag{79}$$

the sensitivity expressed in terms of I_G is the following:

$$\frac{\partial I_G}{\partial E_g} = \frac{G_m}{2\left[1 + \frac{b(a + r_G)}{a + b + r_G}\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]}. \tag{80}$$

Using equations (75) and (78) we get

$$S_n = \frac{\partial i_G}{\partial E_g} = \frac{bG_m}{2[a + b + r_G]\left[1 + \frac{b(a + r_G)}{(a + b + r_G)R_{cd}}\right]}. \tag{81}$$

From equations (74), (75), (77) and (81) we get

$$\frac{S_{\max}}{S_n} = n = \frac{R_{cd}(a + b + r_G) + b(a + r_G)}{b(R_{cd} + r_G)}. \tag{82}$$

With equations (76) and (82) it is possible to solve for a and b in terms of the damping resistance R_{cd} and the sensitivity ratio n. In this way we get

$$a = R_{cd}\left(\frac{n - 1}{n}\right), \tag{83}$$

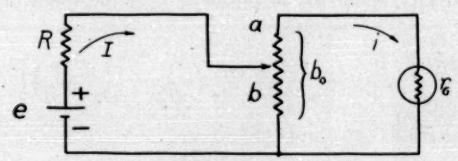
$$b = R_{cd}\left(\frac{1}{n - 1}\right). \tag{84}$$

These equations are very similar to those for the Ayrton shunt in that the values of a and b are independent of the galvanometer resistance. This is, of course, a very real advantage for it makes this shunt a "universal" shunt in exactly the same sense that the Ayrton shunt is a universal shunt. Since the sensitivity equations (32) and (49) for the one tube circuits of Figs. 5 and 6, respectively, are of the same form as equation (74), the constants of the universal shunt for these circuits are also given by equations (83) and (84).

b. Ayrton Shunt and Its Limitations.[14]

The Ayrton shunt can be used in these circuits with a small sacrifice in the maximum sensitivity under certain conditions. These conditions will now be examined somewhat in detail. It is well known that in ordinary circuits

FIG. 9.



Ayrton shunt circuit.

the Ayrton shunt does not reduce the sensitivity in the correct ratio unless the resistance of the associated circuit is considerably higher than the total resistance of the shunt. It will be recalled that the Ayrton shunt is an "L" net work similar to that shown by resistances "a" and "b" of Fig. 9.

The resistances a and b are related to the nominal sensitivity ratio N and to each other according to the following equations:

$$a = b_o\left(\frac{N-1}{N}\right), \tag{85}$$

$$b = \frac{b_o}{N}, \tag{86}$$

$$a + b = b_o. \tag{87}$$

[14] Razek and Mulder, *J. O. S. A. and R. S. I.*, **18**: 390 (1929), suggest using an Ayrton shunt but do not discuss its limitations.

When $N = 1$, we have $a = 0$; and $b = b_o$ which is the total resistance by which the shunt is specified in commercial catalogs. This resistance is usually determined by the external critical damping resistance of the galvanometer. From the simple equations of the circuit shown in Fig. 9 the true sensitivity ratio n corresponding to the nominal ratio N can be worked out. The galvanometer current for any setting is

$$i_G = \frac{eb}{(a + b + r_G)R + b(a + r_G)}, \tag{88}$$

and when $a = 0$ and $b = b_o$ we have

$$i_{G1} = \frac{eb_o}{(b_o + r_G)R + b_o r_G}. \tag{89}$$

Taking the ratio of i_{G1} to i_G as the definition of the true sensitivity ratio n and using equation (86) we get

$$\frac{i_{G1}}{i_G} \equiv n = N\frac{(b_o + r_G)R + b(a + r_G)}{(b_o + r_G)R + b_o r_G}. \tag{90}$$

We can now determine the accuracy with which the true sensitivity ratio is given by the nominal one by the relation

$$\frac{n - N}{n} = y, \tag{91}$$

where y when multiplied by one hundred gives the error in percent.

From equations (85), (86) and (90) we can eliminate a and b and solve for y in terms of the constants of the circuit.

$$y = \frac{Nb_o(b_o + r_G) - N^2 b_o r_G - b_o^2}{N^2 R(b_o + r_G) + Nb_o(b_o + r_G) - b_o^2}. \tag{92}$$

It is easy to show that y is zero for the values

$$N = 1 \quad \text{and} \quad N = \frac{b_o}{r_G}.$$

If $b_o > r_G$, as is usually the case, y has a positive maximum at a value of N between the two for which y is zero, namely, at

$$N = \frac{2}{1 + \frac{r_G}{b_o}}.$$

We see at once that this value of N for which the error is a positive maximum must lie between 1.0 and 2.0. Since the least value of N in most commercially produced Ayrton shunts is 10.0, we find the greatest error when N is large. For large values of N equation (92) reduces to

$$y = -\frac{b_o r_G}{(b_o + r_G)R}. \tag{93}$$

This equation can also be written

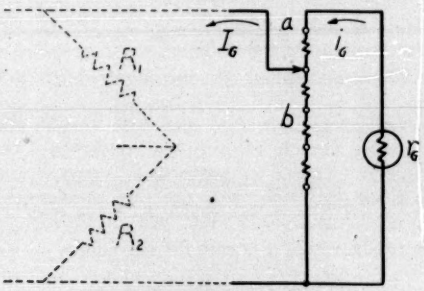
$$y = -\frac{1}{\frac{R}{b_o}\left(1 + \frac{b_o}{r_G}\right)}. \tag{94}$$

Let us take a numerical example in which

$$\frac{R}{b_o} = 2 \quad \text{and} \quad \frac{b_o}{r_G} = 4.$$

Equation (94) shows that the maximum error y will be 10 per cent. when N is large.

FIG. 10.



Circuit with Ayrton shunt for galvanometer control.

Let us consider a case similar to that discussed under the heading "Special Universal Shunt," with the difference that this time we use the Ayrton shunt as shown in Fig. 10. The damping resistance r_d is no longer needed and may

therefore be dropped out. Equation (61) gives the sensitivity of this circuit where I_G is the total current through the shunt b and the galvanometer together and R_G is the resistance of $(a + r_G)$ in parallel with b. We can express the sensitivity in terms of the galvanometer current i_G by using an equation similar to (78) to eliminate I_G. Using these equations and also (87) we get the following:

$$S_n = \frac{bG_m}{2(b_o + r_G) + (a + r_G)b\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}. \tag{95}$$

When $b = b_o$ and $a = 0$, we have the maximum sensitivity S_{max} which is

$$S_{max} = \frac{b_o G_m}{2(b_o + r_G) + r_G b_o\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}. \tag{96}$$

If we define the true sensitivity ratio as before

$$\frac{S_{max}}{S_n} \equiv n \tag{97}$$

and relate it to the nominal ratio N as before by equation (91), we get the following equation for large values of N:

$$y = \frac{1}{\frac{2Z_pR_1}{(Z_p + R_1)b_o}\left(1 + \frac{b_o}{r_G}\right)}. \tag{98}$$

If we set

$$\frac{1}{X} = \frac{1}{2Z_p} + \frac{1}{2R_1}, \tag{99}$$

$$y = -\frac{1}{\frac{X}{b_o}\left(1 + \frac{b_o}{r_G}\right)}. \tag{100}$$

This is exactly of the same form as (94).

The accuracy with which the Ayrton shunt reduces the sensitivity may be of little importance to the user of an amplifier since he can determine the proper correction at any time. However, he may not be satisfied to have the time

required to take readings widely different for the various sensitivities. From an inspection of the circuit in Fig. 10, we see that the damping will be greatest when $N = 1$ and the damping will be least when N is large. This effect may be minimized by making X which is defined by equation (99) as large as is conveniently possible. If we have a value of X at least twice the external damping resistance R_{ed} of the galvanometer, and an Ayrton shunt with a total resistance approximately equal to R_{ed}, then the Ayrton shunt can be used to regulate the sensitivity and the disadvantages which have been pointed out will not be found serious.

If we assume

$$X_{\min} = 2R_{ed} \tag{101}$$

and

$$b_0 = R_{ed}, \tag{102}$$

equation (100) reduces to

$$y = -\frac{1}{2 + \frac{2R_{ed}}{r_G}}. \tag{103}$$

From this we see that if $\frac{R_{ed}}{r_G} = 4$, the maximum error in the Ayrton shunt sensitivity ratio, will be 10 per cent. Equations (99) and (101) serve to determine the minimum satisfactory value of the bridge resistance R_1 which is given by

$$R_{1\,\min} = \frac{R_{ed}Z_p}{Z_p - R_{ed}}. \tag{104}$$

Mention was made above that the use of the Ayrton shunt for a sensitivity control instead of the "special" universal shunt given by equations (83) and (84), usually entailed a small loss in the maximum sensitivity. It is easy to see from equation (62), that if $2Z_p$ and $2R_1$ are large compared with R_{ed} no appreciable loss in sensitivity will result from the use of the Ayrton shunt. On the other hand, if the minimum value of R_1 as defined by equation (104) is used, then the loss in sensitivity in percent will not be greater than $50\frac{r_G}{R_{ed}}$ per cent. It is so easy to construct a

shunt of the kind shown in Fig. 8 that it is advisable to use this network instead of the Ayrton shunt whenever a "permanent" amplifier set-up is being built.

c. Summary on Galvanometer Control.

We have seen that there are two ways of controlling the overall sensitivity of the amplifier to grid voltage. The first required a special type of shunt in which the resistance values depend only on the external critical damping resistance and the desired sensitivity ratio. This shunt when properly arranged gives the computed sensitivity ratios and maintains the damping constant for all settings. The second method suggested depends on the use of a standard Ayrton shunt. Three minor disadvantages are accompanied by the use of this shunt which are (1) the error in the sensitivity ratio, (2) the non-uniformity of the damping, and (3) the loss in sensitivity.

PART 3. SPECIAL INPUT CIRCUITS.

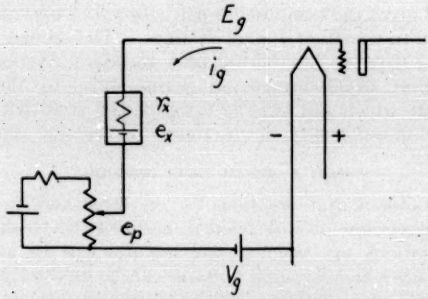
The fundamental equations for the input and the output circuits have now been developed, and in order to show how these relations are used, a few circuits will be analyzed. The circuits which we shall examine can be used for potential and current measurements in high resistance circuits and an attempt will be made to show under what conditions an amplifier can be used to advantage.

A. POTENTIAL MEASUREMENTS.

There are certain problems in which it is necessary to measure a potential originating in a very high resistance circuit. The determination of the acidity of a liquid in terms of a pH measurement with a glass electrode necessitates the measurement of a potential to within about one millivolt while the resistance of the cell may be of the order of ten to a thousand megohms. Although a quadrant electrometer with a sensitivity of 1000 mm. per volt could be used to make this measurement, a galvanometer would need to have a sensitivity of more than 10,000 megs. in order to be used satisfactorily. Where it is necessary to use an instrument which is more rugged than a quadrant electrometer, a galvanometer and an amplifier can be used to considerable advan-

tage. The problem of designing an amplifier for this purpose divides itself in two parts. The first has to do with the input circuit of the tube and is independent of the type of output circuit used. That is, either circuit 5, 6, or 7 or any modification of these can be used without changing the underlying requirements of the input circuit. The second part has to do with the output circuit and the question of *over-all sensitivity* of the system.

FIG. 11.



Simplified input circuit for measuring potentials.

Starting with the input circuit shown in Fig. 11, we see that the following Kirchoff equation must hold.

$$e_x + e_p + V_g = E_g + r_x i_g. \tag{105}$$

Here the resistance of the potentiometer is assumed to be very small compared with r_x. In order to be able to read the unknown potential on the potentiometer directly, the potential e_p should be equal and opposite to that of the unknown e_x.

$$e_x = -e_p. \tag{106}$$

Equation (105) will then be satisfied if

$$r_x i_g = 0, \tag{107}$$

for then

$$V_g = E_g. \tag{108}$$

Of course, equation (107) does not have to be satisfied in the

absolute sense. What is really necessary is that $r_x i_g$ must be small compared with the smallest value of

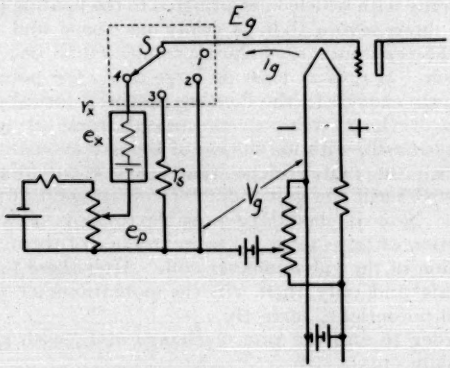
$$e_x - |e_p| = \Delta e \qquad (109)$$

which we wish to measure, for example one millivolt in the case under consideration. Obviously if r_x is very large, i_g must be very small and *we must operate* very close to the floating potential where the grid current i_g can be reduced to a value as small as we need.

a. Case I. Where It is Necessary to Operate at Floating Potential because r_x is Very Large.

If r_x is so large that it is necessary to operate very close to the floating potential (this is the case with most glass

FIG. 12.



Input circuit for measuring potentials.

electrodes if the true potential e_x is to be measured [15]), then a circuit like that shown in Fig. 12 should be used. Here we have the input vacuum tube of circuit 5, 6 or 7. The problem is to measure the unknown potential e_x which originates in a cell or other device having an internal resistance

[15] W. C. Stadie, *J. Biol. Chem.*, **83**: 477 (1929), maintains that only relative values of e_x are necessary for pH measurements with glass electrodes and therefore under certain conditions he does not believe that it is necessary to make $r_x i_g < \Delta e$.

r_x which is very large. The procedure for operating this circuit can be outlined as follows:

1. With switch S in position (1), that is with the grid free, the circuit should be "balanced" to produce no current through the galvanometer in the plate circuit. For example if we use the circuit shown in Fig. 5, the resistance R_a should be adjusted to give no current through the galvanometer, while if we use circuit 7, the grid potential of the dummy tube should be adjusted to bring about the same result.

2. The second step in the process is to move switch S to position (2) and adjust V_g until the galvanometer again reads zero. Now we should be able to move the switch contact from (1) to (2) or back again with no detectable motion of the galvanometer coil in the plate circuit. We have now established V_g = floating potential. This test of the accuracy with which V_g is adjusted to the floating potential may be more severe than is really necessary and for that reason the resistance r_s is shown connected to one point of the switch. If r_s is at least as large as r_x, the potential V_g will be near enough to the floating potential for all practical purposes, if the switch can be moved from (2) to (3) or (3) to (2) at will, with no motion of the galvanometer coil.

3. With the switch in position (4), the potential e_p can be adjusted until the galvanometer coil is again at the zero position. Now it should be possible to move switch S to any position of (2), (3), or (4) from any one of them, without any motion of the galvanometer coil. After these tests have been made and only then will the potentiometer read the unknown potential e_x correctly.

In order to find the rate of change of E_g with e_p, let us differentiate (105).

$$\frac{\partial E_g}{\partial e_p} + r_x \frac{\partial i_g}{\partial e_p} = 1. \tag{110}$$

Using equation (7) and rearranging we find

$$\frac{\partial E_g}{\partial e_p} = \frac{1}{1 + \frac{r_x}{Z_g}}. \tag{111}$$

We see at once that Z_g should be large compared with r_x if

we are to obtain good sensitivity since Z_g is always a positive number at the floating potential. We can combine equation (111) with the other sensitivity equations such as (37), (51) or (63) and we get, using (37),

$$\frac{\partial i_G}{\partial E_g} \times \frac{\partial E_g}{\partial e_p} = \frac{G_m}{1 + \frac{r_G}{R_{cd}}} \times \frac{1}{1 + \frac{r_x}{Z_g}}. \tag{112}$$

Defining the galvanometer response to current [16] as

$$C_s = \frac{\partial \theta}{\partial i_G} \tag{113}$$

(where θ is the angular deflection of the galvanometer coil), we can compute the over-all sensitivity as

$$\frac{\partial \theta}{\partial e_p} = \frac{G_m}{1 + \frac{r_G}{R_{cd}}} \times \frac{1}{1 + \frac{r_x}{Z_g}} C_s. \tag{114}$$

If θ is measured in millimeters deflection at some standard distance, the over-all sensitivity is given directly by equation (114) in millimeters per volt. This expression shows that the over-all sensitivity of the circuit is a very simple function of the tube constants, the galvanometer constants and the resistance of the device in which the potential originates. Equation (114) can be rearranged so as to collect the various factors as follows:

$$\frac{\partial \theta}{\partial e_p} = \frac{G_m}{1 + \frac{r_x}{Z_g}} \times \frac{C_s}{1 + \frac{r_G}{R_{cd}}}. \tag{115}$$

The second factor is a function of the galvanometer only, while the first factor is determined by the vacuum tube constants and cell resistance. It is obvious that if Z_g is large compared with r_x the sensitivity will be proportional to

[16] Current sensitivity (A) is usually given in microamperes per millimeter deflection. $C_s = \frac{10^6}{A}$. Galvanometer sensitivity is sometimes expressed in "megs." C_s = (megohm sensitivity) 10^6.

the mutual conductance G_m and therefore where the cell resistance is quite low the vacuum tube with the highest G_m will have the highest sensitivity. The R.C.A. power tube UX-112-A is a very popular tube having a high mutual conductance. If, however, r_x is ten megohms or more the input impedance as well as the mutual conductance must be considered. Take, for example, $r_x = 50 \times 10^6$ ohms and let us compare the sensitivities which we must expect from a UX-112-A vacuum tube and a UX-222 screen grid tube. The mutual conductances of these two tubes are listed as 1600×10^{-6} mho and 400×10^{-6} mho respectively. The grid impedances, at the floating potential, of tubes of these types have been found to be 12×10^6 ohms for the UX-112-A and 3×10^9 ohms for the UX-222. Considering the first factor of equation (115) which is the part depending on the characteristics of the vacuum tube, we see that the screen grid tube will be 1.3 times as sensitive as the UX-112 and at the same time the UX-222 will be much less sensitive to fluctuations in the power supply.

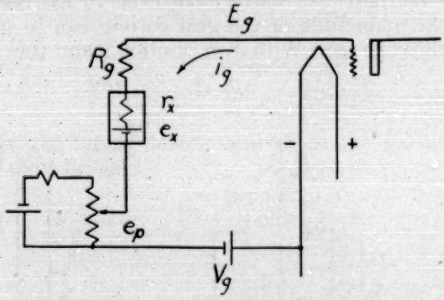
b. Case II. Where It is Possible to Operate with a Negative Grid Impedance because r_x is Small.

When the resistance of the device in which the potential originates is less than one megohm it is not always necessary to operate at the floating potential. The curve shown in Fig. 3, which gives the relation between "C" battery potential and plate current when a high resistance is used in the grid circuit, suggests that a higher sensitivity to voltage can be obtained in an amplifier if we take advantage of the fact that the grid impedance Z_g is *negative* for certain values of grid potential E_g. The use of this method of measurement presupposes that a current of the order of 10^{-8} ampere can flow through the device being measured without altering its properties. If such is not the case we must either select a vacuum tube which has a grid current small enough to cause no change in the device, or else operate near the floating potential and thus keep the current very small. Refer to Fig. 13. This is the simplified form of the circuit which we have in Fig. 14 when switch S is in position 3. The Kirchoff equation corresponding to (105) above is the following:

$$e_x + e_p + V_g = E_g + i_g(r_x + R_g). \qquad (116)$$

Here again we assume that the resistance of the potentiometer is small compared with r_x. By differentiating (116) and

FIG. 13.



Simplified input circuit for potential measurements when r_x is low.

introducing Z_g which is defined by equation (7) we are able to determine the rate of change of E_g with respect to e_p.

$$\frac{\partial E_g}{\partial e_p} = \frac{1}{1 + \dfrac{r_x + R_g}{Z_g}}. \qquad (117)$$

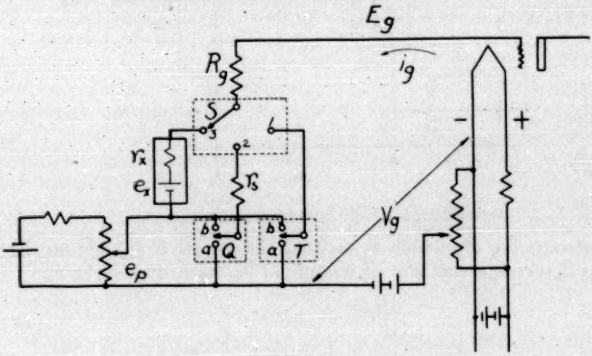
With this equation before us we can turn to the complete circuit of Fig. 14 and examine its operation.

The resistance r_x should be equal to the resistance of the device r_x. The resistance R_g depends on the particular tube used and the grid potential around which we intend to operate. In order to determine the best value of "C" battery potential V_g we set switch S at 2 and switch Q at (a) and the "C" battery potential at some definite value (V_g). The plate circuit should be balanced to produce no current through the galvanometer. With the potentiometer potential e_p adjusted to some small value, throw switch Q to position b and note the galvanometer deflection. This deflection is a measure of the amplifier sensitivity for this particular value of V_g. Now take a new value of V_g and repeat the test, including the balancing of the plate circuit, until the value of V_g is found which gives the maximum

sensitivity. This method of finding the best value of the "C" battery potential for some particular case is obviously a "cut and try" method.

After the best "C" battery potential V_g has been determined, the magnitude of the grid current can be measured in the following way: With S in position 2 and Q in position

FIG. 14.



Input circuit for potential measurements when r_s is low.

a the plate circuit should be balanced to give no current in the galvanometer. With switch T in position b and S in position 1, the potentiometer potential e_p should be adjusted to bring the galvanometer current to zero. Let us designate this potential by e_{po}. The potential e_{po} is now a measure of the "IR" drop brought about by the flow of grid current through resistance r_s. Since r_s is known and e_{po} is determined by the experiment, the grid current can be computed from the Ohm's law relation,

$$i_g = \frac{e_{po}}{r_s}. \qquad (118)$$

Assuming that no alteration in the potential e_x results from the flow of the current i_g, we are in a position to measure e_x.

After the circuit has been balanced with S in position 2

and Q in position a, we switch S to position 3 and adjust e_p to restore the balance. This value of e_p will be equal in magnitude and opposite in sign to the unknown potential e_x.

In order to compute the over-all sensitivity for this circuit we can use equations (37), (113) and (117) to get

$$\frac{\partial\theta}{\partial i_G}\times\frac{\partial i_G}{\partial E_g}\times\frac{\partial E_g}{\partial e_p}=\frac{1}{1+\dfrac{r_x+R_g}{Z_g}}\times\frac{G_m}{1+\dfrac{r_G}{R_{cd}}}\times C_s. \qquad (119)$$

Collecting the factors which depend on the galvanometer, we have the following:

$$\frac{\partial\theta}{\partial e_p}=\frac{G_m}{1+\dfrac{r_x+R_g}{Z_g}}\times\frac{C_s}{1+\dfrac{r_G}{R_{cd}}}. \qquad (120)$$

Since we can operate on a part of the grid characteristic where Z_g is *negative* and we can adjust R_g to make $(r_x + R_g)$ approach the value of $|Z_g|$, the over-all sensitivity of the system can be made very great. The limit of sensitivity is set only by the difficulty of maintaining a steady "zero."

Let us consider a numerical example in which we assume that the unknown potential is of thermo-electric origin and the thermopile has a resistance of 500 ohms, and also that we use the circuit shown in Fig. 5 with a UX-112-A vacuum tube and a Leeds & Northrup 2500-*e* galvanometer. The catalog value of the constants of this galvanometer are the following: sensitivity 3×10^{-9} amp. per millimeter deflection at one meter; external critical damping resistance R_{cd} of 2000 ohms; coil resistance r_G of 500 ohms. The second factor of equation (120) can be computed from these data. We can take $G_m = 1600 \times 10^{-6}$ mho which is the rated value of the mutual conductance for the UX-112 vacuum tube and we can take the grid impedance at the point of inflection of the grid current *vs.* grid potential characteristic as $Z_g = -535$ megohms. This is the value found on a tube of this type and therefore cannot be taken as the average. It is hoped that in the near future measurements can be made on a sufficiently large number of tubes to get some indication as to the probable value of such constants. If such tests are

carried out the results will be published in this Journal. We can make $(r_x + R_g) = 500$ megohms and we get for the over-all sensitivity:

$$\frac{\partial\theta}{\partial e_p} = \frac{1600 \times 10^{-6}}{1 - \frac{500}{535}} \times \frac{1}{\frac{3 \times 10^{-9}}{1 + \frac{500}{2000}}} \qquad (121)$$

$$= 6.6 \times 10^6 \text{ mm. deflection per volt.}$$

The sensitivity is thus 6.6 mm. per microvolt. Remembering that the potential being measured has been assumed to have originated in a 500 ohm thermopile, we see that the sensitivity of this amplifier galvanometer combination exceeds that which can be realized with most D'Arsonval galvanometers. Obviously, if the resistance of the thermopile can be reduced without decreasing its ability to generate potential, the amplifier cannot be used to advantage. On the other hand, if the thermopile can be made to give even higher potentials by a further increase in its resistance the use of an amplifier becomes more and more advantageous. An amplifier used as suggested above will not have a constant zero over any very long period of time, but since the measurement is made by a "null" method (using the circuit of Fig. 14), considerable drift can be permitted without seriously interfering with the measurements.

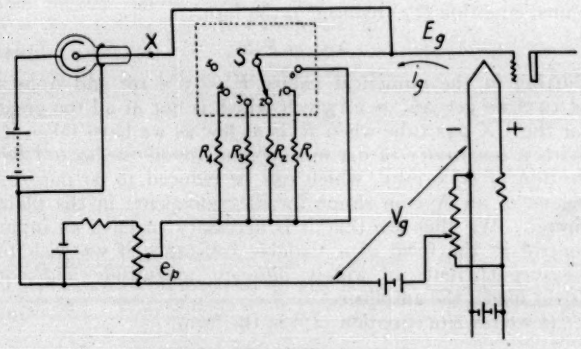
B. CURRENT MEASUREMENTS IN HIGH RESISTANCE CIRCUITS.

Under this head, four important circuits will be discussed. These do not begin to exhaust all of the possible circuits which might be used for current measurements but they illustrate many of the important points to be considered and show the methods by which any special circuit can be analyzed and its operation accurately predicted. Certain special requirements make it desirable to operate some amplifiers with the grid potential very close to the floating potential while in other cases we can realize a greater sensitivity by operating at a potential which is more negative than the floating potential and thus take advantage of the negative grid impedance.

a. Case I. Current Amplifier Operating at Floating Potential. Measurements by Direct Deflection.

The amplifier input circuit shown in Fig. 15 is one which has been used for some years to measure very small photoelectric currents by direct deflection. When there is no photoelectric current, the grid of this amplifier is maintained at the floating potential and the plate circuit is balanced to give no galvanometer current. Since there is no grid current in the input circuit when the grid is at the floating potential

FIG. 15.



Input circuit for photo-electric current measurements.

the "C" battery potential must be equal to the floating potential and the galvanometer zero will be independent of the resistance R_g (i.e., R_1, R_2, etc.). In other words, switch S of Fig. 15 can be moved to any position 0 to 5 without a shift in the galvanometer zero. The deflection of the galvanometer when a photoelectric current is flowing will depend on the value of R_g in a way which will be discussed below. The *only object* in operating this amplifier at the floating potential is to enable us to have a very wide range over which current measurements can be made. It may make this point more clear to illustrate it by means of a numerical example. Equation (17) shows that the change ΔE_g in grid potential brought about by the flow of a photo-

electric current Δi when $R_g = \infty$, is given by

$$\Delta E_g = Z_g \Delta i. \tag{123}$$

If Z_g is 3×10^9 ohms as is the case for the UX-222 vacuum tube, and Δi is 5×10^{-15} ampere, we have a ΔE_g which is 15×10^{-6} volt. With an amplifier sensitivity of 0.4 mm. per microvolt the deflection is 6 mm. Suppose however, we now want to measure a current of 5×10^{-8} ampere. From equation (123) we would get a ΔE_g of 150 volts, which would obviously carry the grid too far from the operating potential. If, however, we use a resistance $R_g = 3 \times 10^6$ ohms, equation (17) reduces to the form:

$$\Delta E_g = R_g \Delta i. \tag{124}$$

Putting in the numerical values $R_g = 3 \times 10^6$ and $\Delta i = 5 \times 10^{-8}$ we get $\Delta E_g = 0.15$ volt which is not at all too great for the UX-222 tube when R_g is as low as we have taken it. With a sensitivity of 0.4 mm. per microvolt we have a deflection of 6000 mm., which can be reduced to 60 mm. by means of an Ayrton shunt (or its equivalent) in the plate circuit. We thus see that it is necessary to have an input control in the form of a variable resistance if we wish to measure currents of widely different magnitude and not "over load" the amplifier.

If we rewrite equation (17) in the form

$$\Delta E_g = \Delta i \left(\frac{1}{\frac{1}{R_g} + \frac{1}{Z_g}} \right) \tag{125}$$

we see the change in potential ΔE_g is equal to the "IR" drop where Δi is the current and the resistance is the external resistance R_g in parallel with the grid impedance Z_g. This assumes that Z_g is constant over the range ΔE_g. That is, we have

$$\frac{1}{R} = \frac{1}{R_g} + \frac{1}{Z_g}, \tag{126}$$

where R is the "effective" resistance which must be used to compute the change in grid potential as

$$\Delta E = \Delta i R. \tag{127}$$

It is by means of this factor R that we control the sensitivity of the input circuit.

Suppose that we wish to reduce our sensitivity and therefore our value of R to some smaller value R'. Let the ratio of these effective resistances be n.

$$\frac{R}{R'} = n. \tag{128}$$

For any value of n and any value of R we can compute the value of R_g which is necessary from equation (126). The highest value of R is had when $R_g = \infty$ for then $R = Z_g$. Let us take the case in which

$$R' = \frac{R}{n} = \frac{Z_g}{n} = \frac{1}{\frac{1}{R_g} + \frac{1}{Z_g}}. \tag{129}$$

Solving equation (129) for R_g we get

$$R_g = \frac{Z_g}{n - 1}. \tag{130}$$

From this equation we can easily compute a series of values of R_g in terms of a given Z_g for any series of sensitivity ratios n_1, n_2, etc. For example with $Z_g = 3000 \times 10^6$ ohms we get

n	R_g
1	∞
3	1500×10^6
10	333
30	103
100	30

The adjustment and operation of this amplifier will now be described assuming that Fig. 15 represents the input circuit to the balanced bridge amplifier shown in Fig. 7. With switch S in position 5 and the photo-cell circuit open at X the grid potential of the dummy tube should be adjusted to bring the galvanometer to "zero." With $e_p = 0$, turn

switch S to position 0 and adjust V_g until the galvanometer again returns to the "zero" position. It should now be possible to turn switch S to any position 0 to 5 without causing a deflection of the galvanometer. If this is not true it is an indication that the grid lead is not properly insulated and protected by shields and guard rings to keep out extraneous currents. If the leakage through the photoelectric cell is smaller than the least current which can be measured with this particular amplifier circuit, then the connection at X can be made without causing a shift in the "zero" as the switch is turned from 0 to 5.

The potentiometer is used to determine the sensitivity of the amplifier to grid potential. With the switch S at 0 the galvanometer deflection can be measured as a function of the potentiometer potential e_p. It may be found that the response will be linear for values of e_p as great as $\pm$ 0.5 volt or more depending on the vacuum tube used. It *does not follow* that the response to photoelectric current will also be linear for the same range of galvanometer deflection. This is due to that fact that the grid current is not a linear function of the grid potential for wide variations in grid potential. For this reason it is well to determine the grid current characteristic by turning switch S to position 1 or 2 and again measure the galvanometer deflection as a function of the potentiometer potential.

This time the potential required to cause a given galvanometer deflection will be greater by an amount Δe_p than it was when switch S was in position 0. This difference in the two potentials, is due to the grid current and is equal to the "IR" drop of this current in the resistance R_g. R_g is equal to R_2 when the switch is in position 2. If we know R_g we can find the grid current i_g corresponding to each value of e_p since

$$i_g = \frac{e_p' - e_p}{R_g} = \frac{\Delta e_p}{R_g}. \tag{131}$$

Where e_p' and e_p are the potentiometer potentials necessary to give the same galvanometer deflection with and without the resistance R_g respectively. Since the grid potential is given by

$$V_g + e_p = E_g, \tag{132}$$

the grid current characteristic as a function of grid potential E_g can be mapped out accurately.[17] This characteristic will usually be of the form shown by the solid line of Fig. 2.

It was stated above that even though the galvanometer response might be a linear function of the grid potential E_g, it does not follow that the galvanometer response will be a linear function of the photoelectric current. Since this non-linearity becomes greater as the resistance R_g increased, let us consider the case in which $R_g = \infty$, that is, the case when the switch is in position 5. We observe that the galvanometer deflection is a function of the light falling on the photo-cell and for each deflection of the galvanometer we can determine the grid potential from the calibration taken with the switch in position 0. With the photo-cell connected as shown the grid will become more and more negative as the light intensity is increased. The amount that it becomes negative will be determined entirely by the grid current characteristic and the photoelectric current will be exactly equal to the grid current for the corresponding grid potential as determined above. This result comes from equation (14) which may be rewritten as follows:

$$i = \frac{E_g - V_g}{R_g} + i_g, \qquad (133)$$

since the first term vanishes when R is infinite. As long as the photoelectric current does not exceed the grid current at the hump of the curve at H of Fig. 2, the photoelectric current will be definitely related to the galvanometer deflection, but the moment the photoelectric current exceeds this value a sudden transition will occur and the grid will become very negative and the galvanometer deflection will increase tremendously even though the light intensity increase is very small. From equation (133), the amplifier calibration and the experimental determination of the grid current *vs.* grid potential curve, we can compute the relation between galvanometer deflection and photoelectric current.

We have gone into this question of the non-linearity of response caused by the flow of grid current at some length, because it enters in such a subtle way that it is very easily

[17] See footnote 1.

overlooked and may cause serious misinterpretations of results if it is not taken into consideration.

b. Case II. Current Amplifier Operating at Floating Potential. Measurements by "Null" Method.

If we use the potential e_p from the potentiometer in Fig. 15 to restore the galvanometer coil to its zero position, the problem of the non-linearity of response no longer plays a part. A new difficulty appears, however, owing to the fact that, as R_g is increased beyond a certain value, the percentage uncertainty in the setting of the potentiometer increases very rapidly. An analytical expression which shows this effect can be derived in the following way.

When we are using the potentiometer as suggested, the Kirchoff equation which we must use instead of (14) is the following:

$$E_g = V_g + e_p + R_g i - R_g i_g, \tag{134}$$

where V_g is constant and equal to the floating potential. When the e_p is adjusted to bring about a perfect balance of the amplifier $e_p + R_g i = 0$; $i_g = 0$ and $E_g = V_g$ = floating potential. In every practical case there is a definite change in E_g (call it δE_g) which produces the least detectable swing of the galvanometer coil. This is a function of the galvanometer sensitivity and the circuit constants. Let us give e_p an arbitrary variation which is just sufficient to produce a change δE_g in the grid potential. We find from equation (134)

$$\delta E_g = \delta e_p - R_g \frac{\partial i_g}{\partial E_g} \delta E_g. \tag{135}$$

Making use of the definition of grid impedance in equation (7) and rearranging we see that

$$\delta e_g = \delta E_p \left(1 + \frac{R_g}{Z_g} \right). \tag{136}$$

From this equation we see which factors control the uncertainty in the setting of the potentiometer and we can calculate the percentage uncertainty as follows:

$$\frac{\delta e_p}{e_p} 100 = U \text{ percent} = 100 \frac{\delta E_g}{e_p} \left(1 + \frac{R_g}{Z_g} \right). \tag{137}$$

Hence, if the sensitivity to photoelectric current is increased by increasing R_g, the percentage uncertainty in the potentiometer setting will change very little as long as $\frac{R_g}{Z_g}$ is less than unity. On the other hand, if Z_g is positive and $R_g > Z_g$, the percentage uncertainty in the potentiometer setting increases with increasing R_g practically as fast as the sensitivity and therefore accomplishes nothing.

c. Case III. Current Amplifier Operating with a Negative Grid Impedance. Measurements by Direct Deflection.

For certain applications it is unnecessary to measure currents extending over a very wide range of intensity with the minimum of adjustment. Very high sensitivity can be had by operating with a fixed resistance R_g and adjusting the "C" battery potential so as to work over that part of the grid current characteristic where the grid impedance is negative. The circuit shown in schematic in Fig. 15, can also be used for this discussion. In the previous section a method was described by which the grid current characteristic of the tube could be measured when the amplifier was balanced with the grid at the floating potential. The procedure in that case was definite because the floating potential could be determined without knowing the entire grid current characteristic. It will appear in the discussion to follow that there are certain advantages to be had if the potential V_g is adjusted so that the grid potential about which we operate is located near the point of inflection of the grid current *vs.* grid potential characteristic. Owing to the fact that the grid current characteristic depends on the conditions in the plate circuit, it is not easy to set up the procedure for determining the point of inflection. The writer has found by experience that the following method leads to satisfactory results. With switch S in position 0 and V_g adjusted to a potential which is 2.0 to 3.0 volts negative with respect to the filament, the plate circuit is balanced to give no current through the galvanometer. This is done by adjusting the potential of the grid of the dummy tube when the two tube bridge circuit is being used.

With the switch still at 0 we should determine the

galvanometer deflection corresponding to a series of potentials e_{p1}, e_{p2}, etc., for both positive and negative values of e_p. If we turn the switch S to position 1, 2, 3 or 4 we can find new values of e_p which will give exactly the same series of galvanometer deflections as before. Assume that we select position 2 and let these new values be e_{p1}', e_{p2}', etc., where e_{p1}' and e_{p1} are the potentiometer potentials which give the same galvanometer deflections with the switch in position 2 and position 0 respectively. These differences $(e_{p1}' - e_{p1})$, etc., measure the "IR" drop in the resistance resulting from the flow of grid current through it. Obviously these differences divided by the resistance R_2 give the corresponding grid currents

$$\frac{e_p' - e_p}{R_2} = i_g. \tag{138}$$

We can add V_g to e_p to get E_g and plot i_g as a function of E_g. This is the grid current *vs.* grid potential characteristic. The reciprocal of the slope of the tangent to this curve drawn through the point $E_g = V_g$ will give the input impedance to the grid when operated under these particular conditions.

If this circuit is to be used for direct deflection measurements, it is important that the value of E_g around which we intend to operate be located on the straight part of the characteristic, that is, near the "point of inflection" of the grid current characteristic. In case the value V_g which was arbitrarily chosen was not close enough to this best value of E_g, a new value of V_g should be taken and the above series of measurements repeated.

In order to test the linearity of response of the system as a whole, two sets of measurements of the galvanometer deflection as a function of potentiometer potential must be made. The first of these is made with the V_g set equal to the value of E_g around which we intend to operate and with the switch at position 0 while the second is made with the switch in position 2, for instance, and with a new corresponding value of V_g. This new value of V_g is found by balancing amplifier with the switch at 0 by means of the dummy tube and then with the switch at 2, V_g is readjusted to

bring the amplifier back to balance. This readjustment is necessary in order to correct for the fall in potential in the grid resistance due to the flow of grid current.

We must now measure the galvanometer deflection as a function of potentiometer potential with the resistance in the circuit and the new value of V_g. The range of galvanometer deflection over which *both* of these calibration curves are linear is a measure of range over which the amplifier as a whole will respond linearly to photoelectric current with this particular value of R_g. It is very important to make both of these tests because it is possible to have a *non-linear* response to photoelectric current and yet have one or the other of these tests indicate a linear response.

The over-all sensitivity using this input circuit and the two tube balanced bridge amplifier of Fig. 7 can be calculated from equations (17), (63) and (113).

$$\frac{\partial\theta}{\partial i_G}\times\frac{\partial i_G}{\partial E_g}\times\frac{\partial E_g}{\partial i}=\frac{G_m}{2\left(1+\frac{r_g}{R_{cd}}\right)}\times\frac{1}{\frac{1}{R_g}+\frac{1}{Z_g}}\times C_s. \quad (139)$$

On rearranging equation (139) we get

$$\frac{\partial\theta}{\partial i}=\frac{G_m}{\frac{1}{R_g}+\frac{1}{Z_g}}\times\frac{C_s}{2\left(1+\frac{r_G}{R_{cd}}\right)}, \quad (140)$$

in which the second factor depends only on the galvanometer constants. Since we are assuming that Z_g is negative we see at once that if R_g is less than the absolute magnitude $|Z_g|$ the sensitivity will increase as R_g approaches $|Z_g|$ and becomes infinite when $R_g = |Z_g|$. If R_g is made larger than $|Z_g|$ it will be impossible to operate the amplifier at the grid potential E_g for which $|Z_g|$ was determined.

If we put constants into equation (140) appropriate to the UX-112-A vacuum tube and the Leeds and Northrup 2500-e galvanometer which are $G_m = 1600 \times 10^{-6}$ mho, $Z_g = -1000 \times 10^6$ ohms, $R_g = 900 \times 10^6$ ohms, $r_g = 500$ ohms, $R_{cd} = 2000$ ohms and $C_s = 3.3 \times 10^8$ mm. per ampere at one meter, we get

$$\frac{\partial \theta}{\partial i} = 1.9 \times 10^{15} \text{ mm. per ampere.} \qquad (141)$$

From this we see that one centimeter deflection corresponds to a current of 5×10^{-15} ampere. In order to determine the actual sensitivity of an amplifier of this type it is not necessary to compute it in this way. All that is necessary is to measure the voltage sensitivity $\frac{\partial \theta}{\partial e_p}$ and multiply it by the resistance R_g. The truth of this statement is at once evident from the equations for the voltage and current sensitivities when written as follows:

$$\frac{\partial \theta}{\partial e_p} = \frac{G_m}{1 + \frac{R_g}{Z_g}} \times \frac{C_s}{2\left(1 + \frac{r_G}{R_{cd}}\right)}, \qquad (142)$$

$$\frac{\partial \theta}{\partial i} = \frac{R_g G_m}{1 + \frac{R_g}{Z_g}} \times \frac{C_s}{2\left(1 + \frac{r_G}{R_{cd}}\right)}. \qquad (143)$$

The first equation comes from (63), (111), and (113) while the second comes from (16), (63) and (113). Substituting (142) into (143) we get

$$\frac{\partial \theta}{\partial i} = R_g \frac{\partial \theta}{\partial e}. \qquad (144)$$

d. Case IV. Current Amplifier Operating with a Negative Grid Impedance. Measurements by "Null" Method.

This type of circuit is very well adapted for "null" measurements in which the potentiometer potential e_p is adjusted to keep the galvanometer coil in its zero position. The question of the linearity of response does not enter for the current being measured is given directly by

$$i = \frac{e_p}{R_g}. \qquad (145)$$

From equation (137) we see that the percentage of uncertainty with which we can set the potentiometer e_p *decreases* as we increase the sensitivity by making R_g larger since Z_g has been assumed to be negative.

PART 4. TECHNICAL PROCEDURE.

A few points as to technical procedure are perhaps worth mentioning.

A. SHIELDING AND INSULATION.

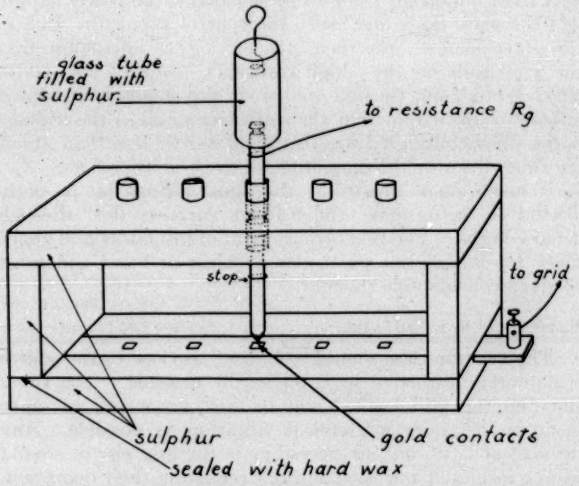
The active grid lead of these circuits should be shielded in all cases where the circuit resistances are very high. The next most important thing to be shielded is the power supply and the plate leads and lastly the control apparatus such as the potentiometer, the rheostats, etc. It is important that the grid leads on the "high resistance" side of the device which is producing the potential or current should be insulated at least to the extent that the resistance of all of the leakage paths when considered together shall not be less than about ten times the absolute magnitude of the grid impedance $|Z_g|$. It is much more important that guard rings be properly located so as to make the leakage currents flow through definite paths. The best arrangement of insulators and guard rings depends on the particular problem at hand and must be given considerable thought.

B. STRUCTURAL DETAILS.

The vacuum tube should be located as close to the source of potential or current as is physically possible. The tubes and also the grid lead should be well supported to make them as free from mechanical vibration as possible. Any switches or keys that are necessary in the grid circuit should be well insulated and mechanically certain in their operation. In the most sensitive circuits used by the author the switch *S* of Fig. 15 was made by imbedding a bar of brass in a cake of sulphur and allowing contacts to drop from above. These contacts were guided by brass tubes also imbedded in sulphur. Small gold tips were set in the movable contacts and small plates of gold were soldered on the brass bar at the points of contact. The sketch in Fig. 16 serves to illustrate the construction. Doubtless there are many other arrangements just as good as this one but this design is given in order to help those whose previous work has been in other fields.

If the amplifier is being constructed for high sensitivity work, great care must be taken to make sure that every junction between wires is soldered. It is even advisable to solder the leads onto the storage batteries used for the filament supply. The control rheostats should be of the best grade and the circuit should be designed with fixed series and shunt resistances associated with each rheostat so that

FIG. 16.



only a small proportion of the current flows through the movable contact. Resistances R_1, R_2 and r_d in the plate circuit should be "wire" resistances. It is usually advisable to insert small resistances in each filament and plate circuit so that the filament and plate currents can be measured with the help of a potentiometer and thus have all circuits undisturbed during the measurement.

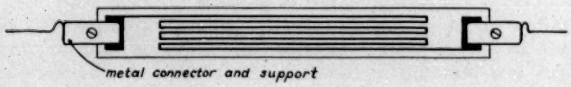
C. CONSTRUCTION OF HIGH RESISTANCES.

The problem of producing high resistances of one or two megohms and more is not an easy one. Many workers in

this field use xylol and alcohol solutions and recommend them highly. The writer has always used "Higgins" India ink resistances and found them perfectly satisfactory. These are made by drawing a long line in the form of a grid as shown in Fig. 17.

A heavy ink line about one millimeter wide drawn on smooth onion skin paper gives a resistance of about one megohm per centimeter. A more narrow line, of course, gives a correspondingly higher resistance per unit length. It has been found advisable to obtain the very high resistances

FIG. 17.



by drawing a very long line rather than a very narrow one.[18] In any permanent set-up these resistances should be mounted in tubes which can be filled with dry air and then sealed or else the tubes should be evacuated and sealed.

The temperature coefficients of three India ink resistances have been measured over the range 23° C. to 45° C. The results are given in the following table:

TABLE III.

$R_t = R_{23}[1 + \alpha(t - 23)]$

R_{23} ohms	α
0.85×10^6	-2.7×10^{-3}
750×10^6	-6.4×10^{-3}
2100×10^6	-6.4×10^{-3}

These resistances have lower temperature coefficients than the one reported by Campbell [19] for xylol-alcohol solutions which was $\alpha = 14 \times 10^{-3}$.

[18] Same experience reported by K. T. Compton and C. H. Thomas, *Phys. Rev.*, **28**: 604 (1926).

[19] N. Campbell, *Phil. Mag.*, **23**: 668 (1912).

APOLOGY.

Although this paper is one of the longest which has been printed on the use of the vacuum tube for D.C. measurements, much has been left unsaid. Many circuits even closely related to those discussed here have not been mentioned. The principal object of this paper has been to formulate in a definite way most of the underlying relationships upon which nearly all of the D.C. amplifiers depend. If these are well understood, the design and operation of an amplifier for any particular purpose becomes a problem which is definite and which can be solved by a straightforward procedure.

THE HEAT CAPACITIES OF ORGANIC COMPOUNDS AT LOW TEMPERATURES. III. AN ADIABATIC CALORIMETER FOR HEAT CAPACITIES AT LOW TEMPERATURES.[1]

BY

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IN 1909 Eucken,[2] working in Nernst's laboratory, devised an apparatus for the measurement of specific heats at low temperatures in which the substance itself acted as the calorimeter. A block of the material in question was suspended in a vacuum to provide thermal insulation, measured quantities of energy were added electrically and the rise in temperature observed by a resistance thermometer or a thermocouple. Corrections had to be made for radiation and other heat leaks. At temperatures below the boiling point of liquid air these were quite small but at higher temperatures they amounted to so much that observations were not very accurate.

This trouble has been corrected to some extent by Nernst and Schwers [3] and to a greater extent by Gibson and Giauque,[4] and by Andrews,[5] and others by using shields of various sorts which keep the temperature of the surroundings in the neighborhood of that of the calorimeter. Obviously, the ideal condition would be when the temperature of the shield is kept at exactly the temperature of the calorimeter at all times. In other words, the process should be adiabatic. The present work is an attempt to do just this, and while certain modifications are perhaps desirable, we believe it shows definitely

[1] From the dissertation submitted by J. C. Southard in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[2] *Physik Z.*, **10**: 586 (1909).

[3] Nernst and Schwers, *Sits. Ber. Preuss. Akad. Wiss.*, 355 (1914).

[4] Gibson and Giauque, *J. A. C. S.*, **45**: 93 (1923).

[5] Andrews, JOURNAL OF THE FRANKLIN INSTITUTE, **206**: 285 (1929).

that it is possible to approach such an optimum condition closely enough to give consistent results. At the same time there are other advantages to be found in this method. It is more rapid, due largely to the fact that it is a continuous process without any tedious wait after heating while the substance comes to thermal equilibrium. The calculation of results is much simpler as there are no corrections to be made for heat leaks, radiation and so on. It is economical, taking about one liter of liquid air for a run and using a standard quart dewar. Furthermore, the great majority of compounds can be obtained pure in the 10 gram samples required here but not in quantities of fifty grams or more which must be generally used for the successful operation of the Nernst type calorimeter.

DESCRIPTION OF APPARATUS.

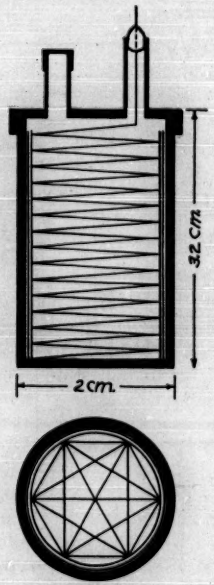
The calorimeter "can" itself (Fig. 1) is made of gold-plated copper and has a capacity of about 8 cc. Two entrance tubes project from the top of it. One is provided with a cap which may be soldered on. It is used for loading and unloading. The other has a copper platinum seal in the outer end of which is fused a soft glass bead. Through this a platinum wire leads to the heating coil. The heating unit itself is a lattice-work of 0.002 inch platinum-iridium wire which is suspended on a mica cylinder. It has a resistance of about 100 ohms at room temperature and about 80 ohms at liquid air temperature. The end of the coil is soldered to the inside of the can which thus serves as the second lead to it.

The can as a whole is suspended by three threads which lead out through holes in the top of the shield, thus enabling one to raise it into position after all electrical connections have been soldered.

The shield (Fig. 2) is a copper cylinder (4 × 11.5 cm.) closed at the top and fitted with a lid at the bottom. It is wound on the outside with a constantan heating resistance in such a manner that the heat is distributed evenly over the whole surface. Heating and thermocouple leads are wound around a tightly fitting inner sleeve several times, the whole soaked with liquid Bakelite, slipped into place and baked at 110 degrees until hard. Any heat leaking down the wires from the outside is thus taken up by the shield. This method

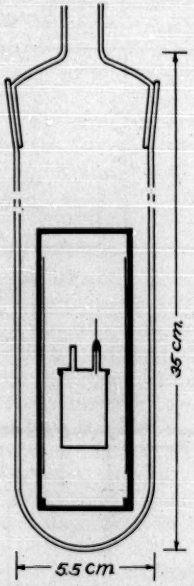
is preferred in this case to the usual one, namely, the use of a heavy block of metal at the top, because the latter causes lags in the change of rate of heating and thus makes it difficult to maintain adiabatic conditions.

FIG. 1.



Calorimeter can.

FIG. 2.



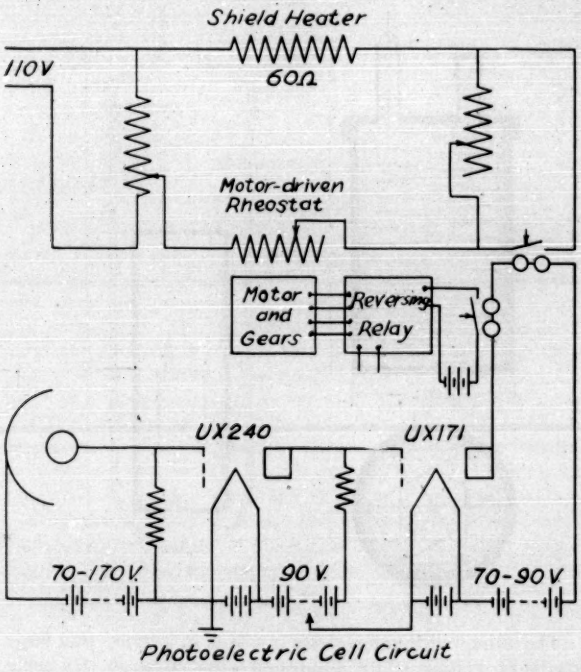
Calorimeter shield and vacuum system.

The shield with calorimeter inside is suspended in a large glass tube connected by a ground glass joint to the usual vacuum system. All the wires are led out through a De Khotinsky seal.

Adiabatic conditions are maintained by the following arrangement. A difference thermocouple between the can and the shield is connected directly to a sensitive mirror galvanometer which operates a photo-electric-cell-relay device

described by us in a previous paper.[6] This operates a reversing relay controlling a motor driven rheostat which regulates the current flowing through the shield heating circuit as shown in Fig. 3. The deflections of the galvanometer which

FIG. 3.



occur in actual practice correspond to a temperature variation of $\pm 0.05°$. The calorimeter may be operated successfully by hand control if such a system is not available.

[6] Southard and Andrews, JOUR. FRANK. INST., 207: 323 (1929).

TEMPERATURE MEASUREMENT.

The temperature was observed by means of a copper-constantan thermocouple which was soldered to the outside of the can at the same point as was the difference thermocouple. This had been compared with a Bureau of Standards platinum resistance thermometer at sixteen points between the boiling point of liquid oxygen and the melting point of ice with the aid of a thousandth degree low temperature thermostat.[7] Comparisons were made at intervals of ten to fifteen degrees, and a temperature e.m.f. scale accurate to 0.01° constructed. Hence little, if any error can arise from this source. This is confirmed by the fact that the fusion temperature of toluene was found to be within 0.01° of that reported by Kelley [8] and that obtained at Leiden,[9] indicating a good agreement among the three temperature scales, in this range, at any rate.

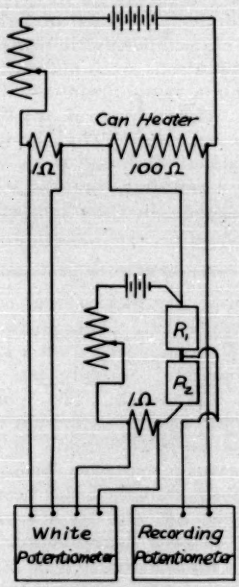
MEASUREMENT OF HEAT INPUT.

The energy input in calories was calculated from the equation $H = 0.23912\ EIt$. The potential drop E across the can heating coil was measured on a Leeds and Northrup recording potentiometer. Actually less than 50 mv. was measured by this instrument. The remainder, amounting to two to three volts was opposed by a constant accurately known e.m.f. as shown in the circuit in Fig. 4. R_1 and R_2 are Leeds and Northrup resistance boxes, and the one ohm coil was checked with one which had been calibrated by the Bureau of Standards. R_1 and R_2 always were set to total 1000 ohms connected by a heavy copper bar and a current of 5 ma. was easily maintained to 0.01 per cent. by checking the drop across the one ohm coil every thirty minutes. This was possible in part because all batteries in the measuring circuits were kept at nearly a constant temperature. The fractional part of the 1000 ohms set in R_1 determined the amount of the known constant opposing e.m.f. in the recording potentiometer circuit. By this means the range of the recorder was tremendously increased and an accuracy of one

[7] Southard and Andrews, *loc. cit.*
[8] Kelley, *J. A. C. S.*, **51**: 2739 (1929).
[9] Com. Phys. Lab. Univ. Leiden, No. 157.

part in 5000 was obtained. In certain cases a L. & N. type "K" potentiometer was employed using two standard cells in series, but the first method gave sufficiently accurate results and not only had the advantage of being semiauto-

FIG. 4.



Heating circuit.

matic but also provided a continuous curve showing the potential drop at any instant.

The current through the can heater was measured by observing the potential drop across a 0.1 ohm coil in series with it on a White potentiometer. By using a 45 volt battery with several thousand ohms in series, a current of about 25 ma. was obtained which varied in a slow and regular manner. The possible accuracy of this measurement was one in several

hundred thousand but in actual practice it was read to one part in ten or fifteen thousand. The time of heating on each observation was about twenty minutes, the stop watches were found to be accurate to about 1/5 second over this period. Hence, the total error due to energy measurement should not be over 0.05 per cent. after a small correction for the leads from the can to the shield is taken into account.

METHOD OF OPERATION.

A sample of material is weighed into the calorimeter can, the latter soldered shut and tested for leaks by placing it in the vacuum system for three or four hours and then re-weighing. The heating leads and thermocouples are then soldered on and the can raised into position in the shield as mentioned above. The outer glass casing is put on, the system evacuated, refilled with hydrogen and a dewar of liquid air brought slowly up around it. In less than an hour liquid air temperature is reached. The system is then evacuated and a current of about 25 ma. started through the calorimeter heating unit causing the temperature to rise about 1/3 degree per minute. The heating of the shield is controlled by hand until its temperature is rising at the same rate as that of the can as is shown by a continued zero deflection of the galvanometer in the difference thermocouple circuit. It is then switched over to the photoelectric cell device. When this has been accomplished, the temperature is read and a stop watch started simultaneously. Since the temperature is rising continually, it is convenient to set the potentiometer dials at an e.m.f. slightly higher than the true e.m.f. at the time and wait, stop watch in hand, until the image from the galvanometer mirror crosses the zero point, at which time the stop watch is started. At the end of about twenty minutes this procedure is repeated except that the watch is stopped. If two watches are used, the second may be started at the same instant that the first one is stopped and so on. When the melting point is reached naturally the rate of temperature increase rather abruptly becomes zero (if the compound is pure) and the current through the shield heating coil has to be quickly readjusted by hand.

The current through the calorimeter coil is read every ten

minutes and the potential drop read from the recorder at leisure. When the run is completed the weight of material is checked, and any variation in the weight of solder used is determined and taken into account in the calculations. The error from this source is probably a matter of a tenth of a per cent. but of course this can be greatly reduced by using a larger calorimeter.

MATERIALS USED.

The benzene used was a "thiophene free C.P." sample which had been further purified by distillation from sodium. The octanol-1 was purchased from the Eastman Kodak Co. and cut on a six foot column vacuum still by G. B. Malone. B.P. 104.5 to 104.6° C. at 25 mm. A freezing point curve showed the M.P. to be − 15.6° C. and impurities to be present amounting to around one per cent. The octanol-4 was prepared and purified by E. E. Reid and co-workers. It boiled at 81.3° C. at 20 mm. and a freezing point curve showed impurities amounting to around 10 per cent. Of course the work on this substance could be used for comparative purposes only. The toluene was a sulphonic acid derivative purchased from the Eastman Kodak Company. It was cut on a three foot fractionating column and boiled over a range too small to be visible on an Anschutz thermometer. No sulfur could be detected on treatment with conc. sulfuric acid. A time-temperature freezing curve showed it to be of extreme purity as did also the lack of any extensive premelting.

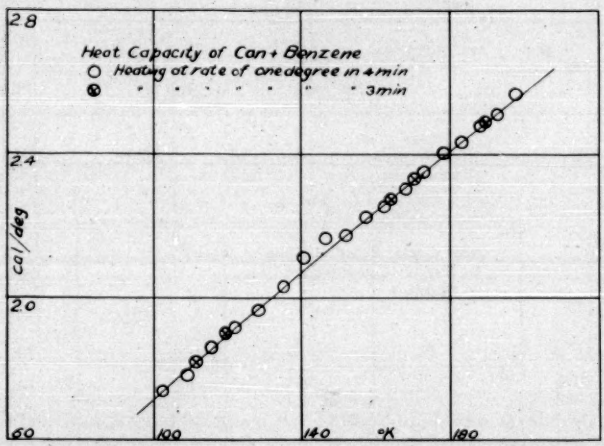
RESULTS.

An adiabatic condition is maintained on an average as is shown by the fact that the heat of fusion was found to be 1584 cal. at 177.94° K. which is within 0.2 per cent. of the mean value found by Kelley [10] using a modification of the Nernst type calorimeter. This value is within the limit of his average deviation. A constant heat leak due to faulty difference thermocouples or temperature control apparatus should show up here quite definitely, in as much as the heat capacity of the empty calorimeter which is found by calibration with a compound of known heat capacity, does not enter into the calculation of the heat of fusion. Linear equations which held quite well for the heat capacities for some distance

[10] Kelley, *loc. cit.*

above and below the melting point were integrated and the observed heat input in the region of fusion corrected in a purely analytical manner. A correction for the resistance of the leads amounts to .9 cal. This may be in error by 0.3 cal. In this case the calorimeter held the fusion temperature $\pm$ 2. μv for over an hour and was in the fusion region for a total of about two hours. The fusion temperature reported was held within 0.1 μv, the limit of the accuracy of observation, for more than forty minutes of this. Obviously, a heat leak over this period of time concentrated, so to speak, into one reading would certainly make itself evident.

FIG. 5.



That equilibrium is maintained within the calorimeter to a satisfactory degree is shown by the fact that a constant rate for temperature increase is attained within one-half minute after the current is turned on. In this time the temperature has risen about 1/6 of a degree. If the whole heat capacity curve were displaced by as much as one-half of this value the error would hardly be noticeable. Also a change of about thirty per cent. in the rate of heating does not displace the curve appreciably as is shown in Fig. 5.

FIG. 6.

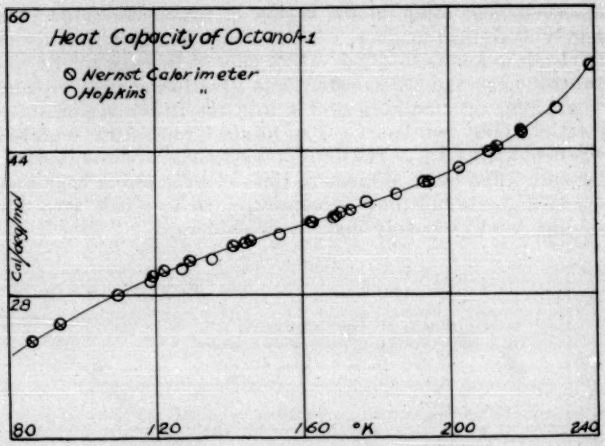
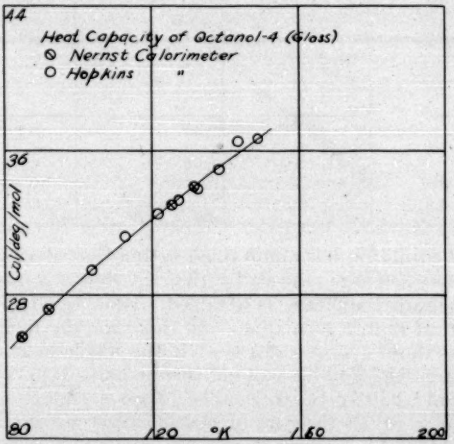
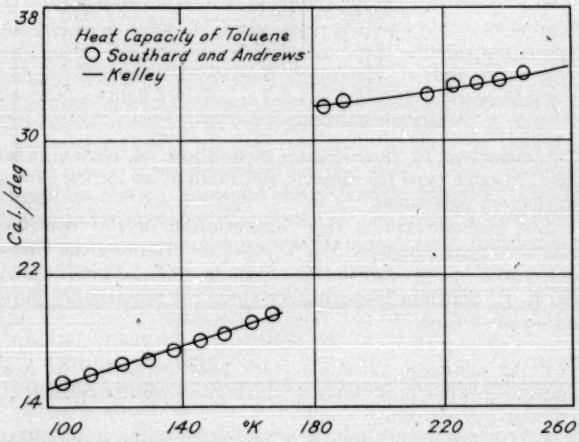

Heat Capacity of Octanol-1
Nernst Calorimeter
Hopkins "
60
44
28
Cal/deg/mol
80
120
160
°K
200
240

FIG. 7.


Heat Capacity of Octanol-4 (Gloss)
Nernst Calorimeter
Hopkins "
44
36
28
Cal/deg/mol
80
120
°K
160
200

To compare the results obtainable with this adiabatic calorimeter with those of the Nernst type, the heat capacities of benzene, octanol-1 and octanol-4 were determined in the calorimeter described by Andrews.[11] These same three compounds were then run in the adiabatic calorimeter. Using benzene as the reference compound, Figs. 6 and 7 show the agreement obtained. The absolute values are probably somewhat high and are not to be taken as final although those for benzene are only about half of a per cent.

FIG. 8.



higher than Nernst obtained.[12] This is largely due to the fact that the steel joint in the vacuum bomb had become somewhat defective and the proper vacuum could not be maintained. The heat of fusion of octanol-1 and octanol-4 could not be accurately determined because of the large amount of impurities present.

Using some recently obtained unpublished values of G. S. Parks for the heat capacity of benzene as a reference com-

[11] Andrews, JOUR. FRANK. INST., 206: 285 (1929).
[12] Nernst, *Ann. d. Phys.* (4), 36: 395 (1911).

pound, lowers these values several per cent. With the calorimeter calibrated on this basis the following values are obtained for toluene. (Fig. 8.)

Heat Capacity of Toluene.

Crystals.		Liquid.	
Temp. K.	C_p, cal./mol.	Temp. K.	C_p, cal./mol.
104.25	15.50	18.38	32.00
112.60	15.90	189.25	32.40
121.97	16.55	217.17	32.75
130.00	16.73	223.14	33.3
137.44	17.38	230.21	33.42
145.23	17.98	237.58	33.6
152.72	18.41	244.79	34.1
160.34	19.11		
167.35	19.61		

Comparison of these values with those of Kelley, who used the same type calorimeter and method as Parks, shows satisfactory agreement.

The authors express their appreciation of the valuable assistance given them by Mr. Teets in determining the time-temperature curves mentioned above, and of the kindness of Mr. R. H. Smith in preparing for them the remarkably pure sample of toluene.

SUMMARY.

1. An adiabatic calorimeter for heat capacities at low temperatures has been devised which gives results consistent to about 1/2 per cent.

2. It is faster and simpler in operation, more economical than the present most used type and applicable to a wider range of organic compounds because of its small size (8 cc.).

3. The heat of fusion of toluene was found to be 1584 calories/mol. at 177.94° K.

4. The heat capacity of toluene was found from 100° K. to 250° K. by comparison with benzene.

SOME RECENT CHANGES IN OUR ATTITUDE TOWARDS THE NATURE OF THE PHYSICAL WORLD.*

BY

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Member of the Institute.

WHENEVER I see a popular or semi-popular book on atoms, electrons, energy, or distant universes, an old story comes to my mind written by a Russian humorist and purporting to be an extract from an ancient chronicle. It runs somewhat as follows: "One evening a famous philosopher was sitting on the steps of his house, contemplating the stars. A passer-by reverently approached him and said: 'Tell me, oh wise man, how many stars are there in the sky?' 'You low-down, ignorant rascal,' answered the philosopher angrily, 'who can encompass immensity?' And the passer-by, honored by a conversation with the sage, and pleased and satisfied with the answer, went on his way rejoicing." True, modern writers on the physical world are more polite and evasive in their statements, and perhaps for this reason the readers are not nearly so well satisfied as the ancient passer-by was.

During the past thirty years the progress in physics, chemistry, astro-physics, and astronomy has been tremendous, perhaps equal to or exceeding that of several preceding centuries. Human curiosity is also greater than ever, and the man in the street is anxious to learn, without spending much time or effort, what this wonderful progress consists in. It is true that in order to understand in general the present state of knowledge in physics, including not only the phenomena but their interpretation as well, one does not have to be a real expert; in fact, with a few exceptions, an expert usually is a narrow specialist in a particular problem and is

* A talk delivered before the Detroit Section of the Society of Automotive Engineers on October 7th, 1929, before the Technology Club of Syracuse on October 14th, 1929, and before the Schenectady Section of the American Institute of Electrical Engineers, on December 6th, 1929.

not even interested in a synthesis of all that is known. Yet, there is a certain minimum of technical knowledge of the phenomena involved and of the mathematical methods of analysis, without which a general story or interpretation of the modern progress in physics is empty and meaningless. It is precisely this minimum of knowledge of the elements that the general reader usually lacks and is unwilling to acquire, making baby talk on the part of even such great writers as Eddington necessary, and leaving the reader exactly where he was before, only more humble and discontented with himself and with popular writers.

Some years ago there was a tremendous rise in interest on the part of the general public in Einstein's theory of relativity, an interest which suddenly subsided with a feeling of disappointment and ennui. The man in the street, or at least those who tried to feed him on popularized relativity, learned three things, to wit: (1) The very special phenomena which had led Einstein to his theory were in themselves not clear to an outsider, and frankly he was not particularly interested in them; (2) The mathematical side of relativity was entirely beyond him; (3) The principle of relativity of space and time did not modify or enlarge his cosmogony, metaphysics, or religion; the new abstruse idea remained entirely disconnected from his intellectual and emotional life, and a few scraps taken in were promptly eliminated from his system, like indigestible portions of his meals.

In this short talk of mine it is impossible even to touch upon the wonderful phenomena and unbelievably delicate measurements of modern physics; it is equally impossible to go into the specific problems of generalization and interpretation of these phenomena and the mathematical means so far mustered for the purpose. Therefore, what I have to say must of necessity be couched in quite general terms borrowed, as much as possible, from the every-day experience of my audience.

Perhaps I can explain by a simile the fundamental nature of the problem with which the greatest physicists are at present concerned. Think of some isolated and fairly primitive agricultural and hunting community in which the political, social, and economic relationships have been so well

crystallized over a long period of time that no one can even conceive of the possibility of a different organization of society. Let us call this fictitious country Arcadia. Suppose now that due to the new and improved means of transportation and communication, the Arcadians have suddenly found themselves on the verge of quite active intercourse with one of the most civilized countries in the world, say the United States of America. The first ambassadors from Arcadia, upon their arrival in this country, would probably at once ask for the king and the chief priest, would look for large estates cultivated by slaves, vast hunting grounds, and so forth. Then, after they will have understood the real structure of our society, it will gradually dawn upon them that the political relations at home, far from being universal and divinely ordained, are of very limited application. They will also find that the conditions in this country cannot be described in terms of some equivalent institutions at home (even by overworking the adjective "heap-big") simply because there may be no native equivalents for trusts, filibuster, racket, etc. Ultimately, after much study of this country, some of the visiting statesmen may get a fair idea of the evolution of the modern state out of a primitive community, and in this manner mentally place the laws and customs at home in their proper relationship with the principal manifestations of political and economic life in America.

Similarly, a spectroscopist who measures what happens within the atom, or an astronomer who deals with the most distant visible celestial objects, soon find that some of the phenomena observed cannot be either checked, predicted, coördinated, or explained in terms of the classical laws of physics and mechanics. Yet these classical laws, from the days of Galileo and Newton, have been found to be almost entirely adequate for "medium-size" objects, including our every-day life and the solar system. It seems, therefore, that the size of an object studied, or its distance, has something to do with the laws which it obeys; the expression "as immutable as Newton's laws of motion," while still somewhat effective as a figure of speech, cannot be used in a strictly scientific treatise anymore. Arcadian writers may still use in their essays the expression "as it is impossible to

imagine a country without a king and a chief priest," but after their acquaintance with this country this expression at the most would become a weak figure of speech.

So far, no known phenomenon indicates a sudden change in the nature of the laws which it obeys when the size of the constituent parts is allowed to vary gradually. Consequently, we are forced to assume that the change from one set of physical laws to another is gradual. This is equivalent to saying that we really do not know yet the most general laws, but that these unknown laws, *within certain limits*, become almost identical with this or that law deduced from a limited range of observation. So a sociologist may write a theory of structure of a modern industrial country, and another may write a theory of primitive hunting and agricultural communities, but to formulate one set of universal laws applicable to the whole human evolution (as was attempted by Spencer), from the ape-like state to Tammany Hall, is a stupendous task not yet successfully achieved.

If I use sand merely for ballast, in boxes or bags, the size of grain and the purity of the material are of no particular interest to me; in fact, I do not even think of individual grains and interstices. But if I use very minute quantities of sand, or mix it with cement, detailed properties of individual grains and impurities become of paramount importance. In the light of modern physics, classical scientists handled matter and energy like sand in big closed boxes the contents of which was unknown to them; the laws which they discovered were "bulk" laws; no wonder that these laws do not apply to small amounts of the contents of those boxes. On the other hand, our solar system is like a small grain of sand compared even to our own galaxy, not to speak of remote galaxies. So the time-space relationships deduced from a microscopic study of such a single grain need not necessarily hold true for the whole mountain from which the grain of sand was obtained. This is how Newton's laws of motion appear only as a first approximation in the theory of relativity.

A Greek philosopher taught that earth consisted of earth particles and water of water particles. I suppose, many a passer-by went on his way rejoicing after having heard such a gem of wisdom, but I doubt if at the present time a naive

theory of this kind would satisfy even a bright primary-grade pupil. The chemistry of a generation ago reduced all existing substances to atoms of some 92 elements and their combinations; modern physics has further reduced all these atoms to not more than three constituents, namely portions of hydrogen and helium atoms, and electrons, in various combinations. The end of the subdivision is not nearly in sight, because each of these constituents has to be endowed with quite a complex structure of its own in order to account for some of the wonderful and manifold physical manifestations known. However, we have already penetrated well into the regions of such small distances that classical laws hold no more.

I can take a small piece of gold and keep on subdividing it by delicate instruments, always obtaining gold of the same density and of the same other physical and chemical properties. It does not make any difference to me whether gold is continuous or consists of discrete particles; it acts like a continuous solid. When, however, I reach such thin sheets that they become transparent to ordinary light and transmit electrons as through a sieve, I become vitally interested in the fine structure of the material. Mathematically speaking, it becomes necessary to assume a discontinuous structure. We now deal with phenomena in the interpretation of which one cannot begin with the favorite classical phrase: "imagine an infinitesimal parallelopiped of dimensions dx, dy, dz." It is like imagining such a parallelopiped in a crate of grape fruit. What you get depends on whether the dimensions of your parallelopiped are of the order of inches or of thousandths of an inch.

A similar situation exists with respect to energy, at least radiant energy. To a classical physicist, light and radiant heat were infinitely subdivisible, and rays could be imagined of any intensity, down to zero. But for at least a quarter of a century, certain phenomena, such as black-body radiation and the photo-electric effect, defied an explanation, except on the basis of a coarse-grained structure of energy. The same assumption underlies modern spectroscopy and some other branches of physics. Thus was the so-called quantum theory of energy originated which is a counter-part of the atomic

and electronic theory of matter. Here again the smallest element, or quantum, of energy (a photon) had to be endowed with various properties before an adequate explanation of even one particular group of phenomena has been reached, and the end of hair splitting is nowhere in sight.

The situation is further complicated by the fact that the minutest known particles of matter, electricity, and radiation do not behave in distinctly different manner from each other on all occasions, but sometimes electrons behave like "wavicles," whereas quanta of radiation behave as if they were little chunks of something more material than ether waves. We may yet have to imagine an entity more general than either matter, electricity, or radiation, of which entity these three are but particular manifestations. Twenty factory watches differ from one hundred similar watches in quantity only, and one watch differs from twenty watches in quantity only. But the elements of a watch, entirely taken apart, differ from a going watch in kind, and cannot be described in terms of a going watch. So it seems as though modern physics has reached a stage of watches taken apart, and not only a new terminology but a new point of view has become indispensable.

Man has an irresistible tendency to associate new things with old: When I meet a new acquaintance, I try to remember him by saying to myself that he looks somewhat like my uncle John; islands and peninsulas have been named after familiar objects with which they have only a remote resemblance. Engineering terminology is full of expressions like wing nuts, choke coils, etc. borrowed from common life. Probably our Arcadians, after their return to their native country would describe our president as a benevolent king, skyscrapers as big huts, and the censor of foreign books in the customs' service as the chief priest sent from heaven. It will take them many years to change their point of view and to devise a terminology adequate to describe this country in independent terms. Similarly, modern physicists started at first to interpret intra-atomic phenomena in a macrocosmomorphic way. An atom emits waves of radiation, so that something must be jerking within it. The nearest approach to such a restlessly moving aggregate in the big world is

the solar system. So the physicists evolved a central nucleus within the atom and a number of electrons revolving around it like planets. Difficulties arose with this interpretation, but they bravely struggled on, adding attribute upon attribute, and special "laws" galore, to make the theoretically predicted results check with experimental data, at least to the right order of magnitude.

Finally, the whole planetary structure of the atom became too complex for a mathematical treatment, or even as a picture, and the special "laws" appended to the picture were in a glaring contradiction with the big world of actual experience. The picture of the atom became a hybrid between some elements taken from the macrocosm, and others tentatively added as working hypotheses. As soon as the faith in the possibility of constructing the microcosm entirely out of the elements of the macrocosm has been shaken, a few bold spirits undertook the task of wiping out the whole clumsy structure and began to paint odd futuristic pictures of the atom, electron, quanta, chemical affinity, and other fundamental concepts, without any reference to the big world. Evidently, such pictures by assumption cannot contain cogwheels, connecting links, orbits, and other crude elements borrowed from every-day experience. Therefore, the new theory in its first steps has become entirely mathematical. We must not talk of definite "things" within the atom, or even of definite physical attributes known only from observations in bulk. Instead we are offered a fundamental property of "something ultra-microscopic" (do not ask what), expressed in the form of a new-fangled mathematical function, which, after being integrated over an appreciable quantity of matter or energy, gives results in fair accord with actual measurements. If this function can be successfully applied to a variety of problems, there must be something fundamentally sound in it; the function must therefore express something intrinsically extant in the atomic world, although we cannot express it as yet in words, for to do this would be to go back to the picture theory of revolving spherical electrons, jelly-like ether, and other childish concepts.

I insist that the recent reaction in physics from the picture theory to the mathematical-function theory is not only

natural, but unavoidable. No matter where man tries to interpret nature, he first assumes the simplest possible relationships from his point of view; an infinite resourcefulness of nature becomes clear to him much later. The earth is at first flat with the sun going around it; then the earth is grudgingly allowed to become spherical; still more grudgingly, ellipticity is admitted. Finally it is agreed that the shape of the earth is that of a geoid, which simply means that the earth looks like itself, has a shape of its own. The new tendency in physics is purely ideological; it is only because of the complexity of the abstract ideas involved that they are represented in the short-hand form of mathematical formulas and new mathematical operations. The great advantage is that later any story that seems plausible can be written, or a picture drawn, around these formulas and operations. Take a simple equation, $ax + b = c$, where x is an unknown quantity. This equation can be made to represent the story of a merchant who bought an unknown number of yards of cloth, of a train which started at an unknown instant from a station, of a man who pumped water at an unknown rate, and what not. In regions unknown a purely formal mathematical approach is the safest.

Another way out of being forced to describe too minutely a phenomenon about whose sub-microscopic details and mechanism we know but little, is to use the statistical method. I have no idea how long John Smith is going to live, but given sufficient data I could tell what his expectation of life or the probability of his dying within the next five years is, what the average income of men of his age and education is, etc. This information is not going to tell you when John Smith is going to die nor what his income actually is, but somehow you will feel that you have obtained some useful and essential data about the man. The statistical method in physics was first applied to perfect gases, about the middle of the nineteenth century, giving rise to the so-called kinetic or dynamical theory of gases. With the statistical method, the mathematical physicist does not attempt to specify the position and the velocity of each of millions of particles of a gas, per cubic centimeter, but only computes the probability of a particle having a velocity or kinetic energy between

some two given limits. With a very large number of particles, this probability becomes identical with the actual number of particles endowed with that velocity or kinetic energy. For example, if the probability of being involved in an automobile accident in a large city, within a month, is one-hundredth of a per cent., it simply means that actually one out of every ten thousand in that city figures in an automobile mishap each month.

The achievements of the classical kinetic theory of gases in checking and predicting phenomena quantitatively were so satisfactory that it was only natural to extend the statistical method to new problems, involving not only molecules of a gas, but quanta of radiation and electrons as well. However, the early results were far from satisfying, and a possible way out of the difficulty (not a complete solution as yet) has been found in the realization of the fact that there are different kinds of statistics, that is, different methods of computing a probability. We are gradually approaching the situation in statistical mechanics wherein statistical methods are becoming so flexible that almost any desired "bulk" law could be duplicated by choosing suitable assumptions as to single and compound probabilities of the elementary individual events. Let us hope that we shall not go in this respect as far as we have in politics and expert accountancy, where statistics are sometimes made subservient to the will to believe.

The differences between the probability computations in the old and the new statistics may be reduced to the following three: (*a*) In the classical theory, the amounts of kinetic energy ascribed to various particles varied continuously; in the quantum theory these amounts are assumed to vary in small but finite steps. (*b*) In the classical theory each molecule was supposed to have a distinct individuality, so that interchanging the positions or the velocities of two molecules gave a new distribution or arrangement. In the new statistical mechanics, the molecules, electrons, or photons are assumed to be indistinguishable from one another, so that interchanging two elements does not give a new arrangement which would increase the total probability of that particular distribution. (*c*) In the classical theory of gases, the particles and their energies were assumed to be entirely independent of one

another, whereas in the new statistics assumptions are made as to the maximum number of elements which can have a specified amount of energy each, so that different groups of particles are forced to occupy different "levels" of energy.

To obtain a general idea of the nature of statistical or probability problems in modern physics, consider the following simple case: There is a group of 15 compartments or cells, corresponding to 5 units of energy, in the sense that any particle which is placed in one of these cells become thereby endowed with 5 units of energy. There is also a group of 20 cells corresponding to 6 units of energy. The total number of available particles to be placed in these cells is 45 and the total combined energy of the particles is 250 units. What distribution of particles in the cells permits of a maximum number of individual arrangements, on the supposition that the cells are distinct whereas the particles are indistinguishable from one another? In actual problems on equilibrium of particles of gas, an assembly of electrons, or black body radiation, the number of groups of cells, the number of cells per group, and the number of available particles are very large, running into millions. Moreover, other conditions are imposed, for example, (*a*) the total number of photons is not a constant given quantity as the number of particles is above; (*b*) not more than one particle may occupy a cell; (*c*) the presence of a particle in a cell increases or reduces its "attractiveness" for another particle by a given fraction. In this manner, one obtains the so-called Bose-Einstein and Fermi-Dirac statistics, and their later modifications and generalizations.

The distribution which gives the largest number of individual distinguishable arrangements is the most probable distribution and the one about which the actual state of the aggregate fluctuates. Moreover, knowing the probability of various other arrangements, the average state of the aggregate (and consequently the magnitude of fluctuations) can also be determined. As stated above, the new statistical methods are quite flexible and can be modified and generalized as actual need arises. In a sense, they have an element of a picture theory in them, for example, when it is stated that the presence of a photon in a cell increases the probability

of another photon joining it in the same cell. However, the picture remains largely mathematical, without describing the shape of the hooks by means of which one photon attaches itself to another.

Now, even if we interpret all the sub-atomic entities in a mathematical form incapable of visualization, and even if we do the same with the vast spaces of the universe where our ordinary Euclidean relationships hold no more, this will not do away entirely with the bias of the medium-sized world in which we live. For, after all, even our mathematics has been entirely evolved in this inescapable workaday world of ours. To be strictly logical, we should not even use the addition and multiplication signs in the proposed formulas for the sub-atomic and ultra-galactic worlds, because these mathematical operations savor of the earthly occupations of barter, walking, measuring fields, or counting slaves. But here we are almost reaching the limits of the workings of the human mind, viz., the fundamental categories in which it can reason at all. Not that we have reached the highest concepts that the human mind is capable of; compare crude reasoning of a savage with that of our most advanced analytical scientists and synthetic philosophers. Yet, the progress in the very categories of thinking must of necessity be slow and only possible at large intervals, when a new genius arises among us. The purely formal or experimental progress in science is continuous so long as there are talented and devoted men working on a given problem.

Summarizing, I would express my opinion upon the changing point of view in physics as follows: (1) Within the last thirty years a very rapid progress in the technic of measurements has extended the data of physics to extremely small quantities and the data of astronomy to enormous distances and unimaginably large intervals of time. Moreover, many of these data are correct to a high degree of accuracy. (2) The human mind is never satisfied with a mere accumulation of data and always wants to interpret them in terms of some general laws or relationships. The first and natural step was to extend and to adapt the previously known laws of geometry, physics, and astronomy to the newly conquered regions. (3) Since these laws failed

in some cases, especially in the intra-atomic world, a picture-theory of the microcosm was evolved, patterned somewhat after the solar system. Later it became necessary to add numerous details and special laws to the initially simple picture, and even then it did not agree with many experimental data. (4) As a reaction, the picture theory of the atom has given place to a mathematical or ideational theory (wave mechanics) in which the unknown inner workings are expressed by newly devised formulas and operations for which no physical picture is offered. The proof of these formulas lies in the results obtained by applying them to measurable quantities of matter or energy. (5) In astronomy, the picture-theory step was omitted and the generalized theory of relativity has given us directly new time-space relationships, impossible to be visualized, and strictly mathematical in their formulation and application.

I shall not attempt a prophecy as to whether physics will continue to progress and develop at the present unprecedented rate, or whether the progress will slow down. Even assuming that no additional new ideas of importance are to be evolved by the present-day physicists of the first rank and no new scientist of extraordinary vision and skill is to appear say within the next twenty years, it is difficult to conceive of an appreciable slowing down of research in physics. The brilliant and revolutionary ideas hurled at us within the last ten or fifteen years will be sufficient to keep hundreds of talented workers busy for many years to come applying and further developing these ideas in detail. We should be grateful for what has been so generously revealed to us and see to it that the new knowledge has been put to proper use for the benefit of humanity, before expecting more.

ITHACA, N. Y.

THE SOLARIZED LATENT IMAGE OF PHOTOGRAPHIC EMULSIONS.*

BY

A. P. H. TRIVELLI.

INTRODUCTION.

If a photographic plate or film is exposed to increasing amounts of light the developability of the photographic emulsion will increase, resulting in increased density after development. This reaches a limit with continued exposures, called the maximum density. With still longer exposures the developability diminishes and after reaching a minimum it again increases. This is the region of solarization, in which we distinguish, (*a*) a region of the first reversal of the solarization; (*b*) a region of minimum density; and (*c*) a region of the second reversal of the solarization, which is a re-reversal. Reversals of higher order are uncertain. Lüppo-Cramer[1] discovered the probability of their existence, but he admits the necessity of more experimental evidence.

In 1915, C. E. K. Mees[2] wrote: "Any theory of photographic action must attempt to explain the region of reversal." It is the purpose of this paper to analyze already known explanations and to give a tentative hypothesis of the solarized latent image, based in part on experimental investigations carried out in these Laboratories and in part on the previous ideas of other investigators.

Actions similar to those with light are secured with hydrogen-peroxide as observed by Lüppo-Cramer,[3] J. Precht and Otsuki,[4] B. Dombrowsky,[5] and S. E. Sheppard and E. P. Wightman;[6] with ozone as noted by K. Schaum and W.

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* Communication No. 416 from the Kodak Research Laboratories.

[1] *Das latente Bild*, 37: (1911).

[2] Jour. Frank. Inst., 179: 148 (1915).

[3] *Phot. Korr.*, 563 (1902).

[4] *Ann. Physik*, 16: 890 (1905).

[5] *Inaug. Dissert.*, 68 (1908).

[6] Jour. Frank. Inst., 195: 337 (1923).

Braun;[7] with persulfates by Lüppo-Cramer;[7a] with certain organic peroxides in the presence of moisture as shown by B. T. Brooks;[8] with sodium arsenite as observed by W. Bancroft,[9] G. A. Perley,[10] and W. Clark;[11] and with stannous chloride, as noted by G. A. Perley.[12] We assume that W. Clark's statement [13] to the effect that the action of reversal produced by fogging agents is probably due to peptization of the latent image is correct. This restricts the content of this paper to the light action only.

To explain the first reversal of the solarization several theories have been suggested. Solarization being the result of the development of a latent image in this region called the solarized latent image, gives rise to the proposal of two kinds of theories, namely: (*a*) theories which try to explain the solarization as a development effect, and (*b*) theories which try to explain the solarization as a pure latent image effect. Only the best known theories will be critically discussed here.

DEVELOPMENT THEORIES OF THE SOLARIZATION.

I. The theory of R. E. Liesegang [14] states that the silver halide grain in the overexposure region is so surrounded by a small layer of silver that the developer is prevented from penetrating the silver halide grain. The fact that it is possible to obtain the first reversal of the solarization after fixation with physical development as has been shown by J. Sterry,[15] Lüppo-Cramer,[16] and J. M. Eder [17] is in disagreement with this theory. This is worth mentioning since F. W. T. Krohn [18] proposed this theory as a probable explanation.

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[7] *Z. wiss. Phot.*, **2**: 285 (1904).
[7a] *Phot. Korr.*, 563 (1902).
[8] *Brit. J. Phot.*, **57**: 119 (1910).
[9] *J. Phys. Chem.*, **14**: 292 (1910).
[10] *J. Phys. Chem.*, **14**: 685 (1910).
[11] *Brit. J. Phot.*, **70**: 717 (1923).
[12] *Loc. cit.*
[13] *Phot. J.*, **64**: 363 (1924).
[14] *Archiv wiss. Phot.*, **2**: 261 (1901).
[15] *Phot. J.*, **22**: 264 (1898).
[16] *Phot. Wochenblatt*, 303 (1909). *Phot. Ind.*, No. 35 and 37 (1917).
[17] *Eder's Handb.; Photochemie*, 293, footnote.
[18] *Phot. J.*, **64**: 369 (1924).

II. The gelatin-coagulation theory of R. Luther [19] regards the cause of the first reversal of the solarization as a coagulation of the gelatin around the grain by the photochemically liberated halogen, which will resist the penetration of the developer to the grain.

It is difficult to picture how a re-reversal of the solarization by prolonged exposure is possible. In addition to this, the theory is in diagreement with the fact that the solarization also appears in gelatin-free silver halide emulsions, such as collodion emulsions, and even on silver halide plates, which are entirely free from any binding material, as was shown by K. Schaum and W. Braun.[20] E. English [21] modified this theory, regarding it as quite possible that this coagulation of the gelatin intensifies the first reversal of the solarization, disregarding the second reversal. This point of view was rejected by Lüppo-Cramer,[22] H. Weisz,[23] E. W. Buchner,[24] and B. Walter.[25] The coagulation of the gelatin by exposures of the magnitude required to give solarization, however, was shown by the present author.[26] To what extent it accelerates the first reversal of the solarization is still unknown.

LATENT IMAGE THEORIES OF THE SOLARIZATION.

The proposed theories of a solarized latent image can be divided in two groups: I. A progression theory; and II. a regression theory. At the present, different outstanding investigators accept one of the two theories. Defenders of the progression theory are: J. Eggert and W. Noddack,[27] H. Arens,[28] H. Scheffers,[29] W. Leszynski,[30] and W. Meidinger.[31]

[19] *Die chem. Vorgänge in der Phot.*, 47 (1899).
[20] *Z. wiss. Phot.*, **1**: 377 (1903).
[21] *Archiv wiss. Phot.*, **2**: 262 (1901).
[22] *Phot. Korr.*, 348 (1901).
[23] *Z. physik. Chem.*, 334 (1906).
[24] *Phot. Korr.*, 234 (1904).
[25] *Ann. Physik*, 4th series **27**: 83 (1908).
[26] *Phot. Korr.*, 29 and 182 (1911).
[27] *Eder's Handb.*, **II, I**: 246 (1927).
[28] *Z. physik. Chem.*, **114**: 337 (1925).
[29] *Z. Physik*, **20**: 109 (1903).
[30] *Z. wiss. Phot.*, **24**: 282 (1926). *Phot. Ind.*, **26**: 425 (1928).
[31] *Handb. phys. Optik*, 50 (1927).

Defenders of the regression theory are: W. Abney,[32] Lüppo-Cramer,[33] M. Andresen,[34] J. M. Eder,[35] C. E. K. Mees,[36] H. S. Allen,[37] and H. Kieser.[38]

T. Slater-Price [39] criticizes both theories and rejects them both.

I. *The Progression Theory of the Solarization.*—The progression theory explains the solarization as a supposed coagulation of the latent image. On exposure a finely divided silver is first produced at the surface of the grain, which is active and is the substance of the normal latent image. On further exposure the silver becomes more compact by heaping together and loses its catalytic properties. In favor of this theory is the fact that in the region of the solarization the total amount of photochemically formed silver does not decrease, but increases in proportion to the total amount of exposure. This has been shown by J. Eggert and W. Noddack,[40] H. Scheffers,[41] and H. Tollert.[42]

Lüppo-Cramer [43] correctly states, however, that it is not so much the total amount of photochemically formed silver that counts in the solarization, as the manner of distribution of this silver over and in the silver halide grain topographically. That light is able to coagulate submicrons was shown by A. Galecki [44] for X-rays, by Th. Svedberg [45] for ultraviolet light, and by P. B. Ganguly and N. R. Dhar [46] for ordinary sunlight, but W. Clark [47] remarks, "Any coagulation

[32] *Phot. J.*, **48**: 318 (1908). *Treatise on Phot.*, Ed. 5, 309.

[33] *Phot. Korr*, 376 and 439 (1907). *Das latente Bild*, 39 (1911). *Phot. Ind.*, **26**: 596 and 960 (1928). *Z. wiss. Phot.*, **25**: 224 and 308 (1928). *Eder's Handb.*, **II, I**: 600 (1927).

[34] *Das latente Lichtbild*, 36 (1913).

[35] *Z. physik. Chem.*, **117**: 293 (1925). *Eder's Handb.*, **II, I**: 585 (1927).

[36] JOUR. FRANK. INST., **179**: 141 (1915).

[37] *Photo-electricity* (1913).

[38] *Z. wiss. Phot.*, **26**: 321 (1928).

[39] *Brit. J. Phot.*, 507 (1925).

[40] *Z. Physik*, **20**: 299 (1923).

[41] *Loc. cit.*

[42] *Z. physik. Chem.*, **140**: 355 (1929).

[43] *Z. Physik*, **21**: 387 (1924).

[44] *Koll. Zeit.*, **10**: 149 (1912).

[45] *Koll. Zeit.*, **6**: 238 (1909).

[46] *Koll. Zeit.*, **31**: 16 (1922).

[47] *Phot. J.*, **64**: 363 (1924).

of the latent image could take place only on each individual grain, and not from one grain to another, so that in this case the developability of a grain would not be reduced, since the size of the reduction nucleus would be increased. It would be expected that, in the presence of such an effective protecting agent as gelatin, coagulation of colloidal particles would be far more difficult of attainment than their peptization."

Lüppo-Cramer [48] has actually deduced that agglomeration increases the rate of development. L. Silberstein [49] has shown the same thing quantitatively, and the observations of F. Toy and of R. P. Loveland with the author agree with his calculations. T. Slater-Price,[50] however, does not regard this as sufficient argument against the coagulation theory, since the observations may be explained by the assumption that the agglomeration is that of amicrons of size insufficient to act as nuclei, to particles sufficiently large to act as nuclei.

H. Scheffers [51] was able to show experimentally with a physical developer and colloidal silver particles in suspension, that with increasing size the rate of increase of the size of the particles first increases and then decreases, because the ratio of the surface to the mass of silver of the particles decreases with increasing size of the particle. This is supposed to show how the increasing size of the speck by coagulation would diminish developability. But in that case it would only shift the difficulty to the explanation of the second reversal of the solarization, which is in disagreement with this explanation of the first reversal of the solarization. H. Arens [52] tried to solve this difficulty by assuming that this second reversal of the solarization does not exist and that there is shown only an increase of density caused by the blackening of the photochemically formed silver. The reality of the second reversal of the solarization is admitted in the experiments of Lüppo-Cramer and E. Stenger [53] on plates with peptized silver bromide gel, but this was regarded as an exceptional case.

[48] *Koll. Zeit.*, **15**: 164 (1914); **28**: 174 (1921).
[49] *Phil. Mag.*, 464 (1928).
[50] *Loc. cit.*
[51] *Loc. cit.*
[52] *Loc. cit.*
[53] *Phot. Korr.*, 443 (1910).

As a matter of fact, in 1907 Lüppo-Cramer [54] showed the reality of the second reversal of the solarization.

This method of subtracting visual photochemically formed densities from densities which were obtained after development is not acceptable. T. Slater-Price pointed this out. The relation between density and the amount of silver is not the same for silver obtained by the printing-out effect and fixing as for silver produced by development.

This same method of subtracting was used by H. Arens to show that halogen absorbers, as sodium nitrite, do not remove solarization. In the presence of such substances the printing-out effect is increased so much that it covers the reversal. The same had already been expressed as a probability by W. D. Bancroft.[55] J. M. Eder [56] has shown conclusively, however, that this explanation does not hold, and that the solarization diminishing and removing action of sodium nitrite is a well established fact. According to Lüppo-Cramer the action of sodium nitrite and other halogen absorbers should increase the solarization, if the coagulation theory were true. But W. Leszynski, admitting that the action of halogen absorbers is unexplained, does not regard this striking phenomenon as evidence against the coagulation theory.

In an investigation of correlations between different photographic characteristics in the normal and in the solarization region of exposures by E. C. Jensen and the author [57] it was stated that, according to the coagulation theory, we may expect to find a correlation between the solarization sensitivity and the average grain size with different photographic emulsions. In our observations we could find no trace of a correlation between these two, but we found a definite correlation between the content of silver iodide of the silver halide of the grains and the sensitivity of the first reversal of the solarization and, according to the investigations of H. Tappen [58] and V. Schumann,[59] this correlation has a

[54] *Photographische Probleme*, 157 (1907).
[55] *J. Phys. Chem.*, 13: 330 (1909).
[56] *Loc. cit.*
[57] JOUR. FRANK. INST. (in press).
[58] *Phot. Korr.*, 365 (1908).
[59] *Eder's Jahrbuch*, 391 (1898).

positive coefficient, *i.e.*, increased silver iodide content increases the first reversal of the solarization. The negative coefficient we found was due to the overlapping and dominating influence of the sensitivity specks, which diminish solarization.

Th. Svedberg and H. Anderson,[60] however, have shown that in the first reversal of the solarization the larger grains solarize sooner than the smaller grains. Preliminary investigations of the author on one-grain layer plates show the same results and are in agreement with the determined covering power of the silver in the solarization region by A. Ballard.[61] Quite different are the observations of H. Scheffers [62] who found in the reversal portion that the percentage grains changed remained practically constant.

This seems contradictory to the results obtained by correlation investigations, but F. F. Renwick and V. B. Sease [63] found by an ingenious method of sedimentation analysis, that the larger grains have a higher content of silver iodide than the smaller ones of the same emulsion. This was confirmed in these Laboratories.[64]

Therefore, in one and the same photographic emulsion containing silver bromo-iodide grains we will find a greater sensitivity of the first reversal of the solarization at the larger grains, because of their higher silver iodide content; but if we take a number of different photographic emulsions we will find the mentioned correlations.

Summarizing, we may say that the coagulation theory of the solarization in its present state is not acceptable on account of the fact that it does not agree sufficiently with a number of phenomena described above.

II. *The Regression Theory of the Solarization.*—F. Kogelmann [65] and H. Luggin [66] point out that it is the silver on the surface of the grain which is of greater importance in the production of developability than the silver within the grain.

[60] *Phot. J.*, **61**: 329 (1921).
[61] JOUR. FRANK. INST., **208**: 731 (1929).
[62] *Loc. cit.*
[63] Colloid Symposium Monograph, **2**: 37 (1924). *Phot. J.*, 360 (1924).
[64] JOUR. FRANK. INST., **203**: 829 (1927).
[65] *Isolierung d. Substanz d. latenten Bildes* Graz., 24 (1894).
[66] *Eder's Jahrb.*, 162 (1898).

According to the concentration speck theory of the latent image as proposed by S. E. Sheppard, A. P. H. Trivelli, and R. P. Loveland [67] and which the author attributes to the probable photoelectric action of a Becquerel cell of amicroscopical dimensions,[68] the light increases the size of the preexisting sensitivity specks on the surface of the silver halide grains to a size that introduces developability. The idea of a nucleus of minimum size is not new and has already been mentioned by J. M. Eder,[69] Th. Svedberg,[70] and F. F. Renwick.[71] It is obvious that the first reversal of the solarization being a diminishing of the developability of the speck suggests a decrease of the size of the speck by prolonged exposure. This led to a dispersion theory of the latent image in the solarization region. That light is able to disperse metals has been shown by Th. Svedberg,[72] but it is quite a different thing to disperse a piece of metal in a liquid or to disperse a colloidal particle adsorbed on a silver halide crystal surface and surrounded by solid gelatin, where there are well established evidences for the coagulation action of light on colloidal particles as has been mentioned above.

F. F. Renwick [73] suggests that the solarization is due to a peptizing action on the latent image "on the part of the later formed chemical products of light action (bromine, hydrobromic acid, *etc.*) with formation of a photo-halide relatively rich in dissolved silver, but almost undevelopable." W. Clark [74] says about this: "Although there is not yet much evidence in support of this explanation, it is clear that the presence of the silver halide, together with the latent image, is necessary for reversal to occur."

In 1915 C. E. K. Mees [75] said: "It seems probable that reversal is due to slow decomposition of the silver bromide and liberation of bromine, which then attacks grains, which had been exposed and contained a nucleus."

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[67] JOUR. FRANK. INST., **200**: 51 (1925).
[68] JOUR. FRANK. INST., **204**: 649 (1927), **205**: 111 (1928).
[69] *Eder's Handb.*, **II, I**: 288 (1927).
[70] *Phot. J.*, **62**: 312 (1922).
[71] *J. Soc. Chem. Ind.*, **39**: 156T (1920).
[72] *Colloid Chemistry*, 62 (1924).
[73] *Loc. cit.*
[74] *Phot. J.*, **64**: 365 (1924).
[75] *Abr. Sci. Pub. Res. Lab.*, Eastman Kodak Company, **I**: 74 (1913/14).

This reduces the explanation to an action on the size of the speck through photochemically formed bromine and the only problem left is to determine the source of the liberated halogen. Lüppo-Cramer does not express himself on the subject of the size of a speck in relation to developability, but speaks in general about colloidal silver as the substance of the latent image. With this more general view, however, he had, long before anybody else, recognized that the explanation of the first reversal of the solarization should be topographically on the basis of a regression. He thinks that the halogen is liberated within the grain and diffuses through the silver bromide crystals to the surface, where it reunites with free silver. In this way it should be quite possible that the total amount of photochemically formed silver in the grain increases continually and that at the same time the silver particles on the surface of the grain decrease in size, diminishing the developability. This explanation, however, was rejected by T. Slater-Price: [76] "It is difficult to see how the process postulated by Lüppo-Cramer could take place. During the exposure the intensity of the light will diminish from the surface to the interior of the grain owing to absorption. The amount of photo-decomposition will also diminish, not only because of the decrease in light intensity, but also because in the interior of the grain the bromine pressure will be greater, tending to reverse the reaction at the surface, where the bromine can be taken up by the gelatin. During the actual exposure there is no *a priori* reason why the free silver at the surface should combine with bromine, which may diffuse there from the center, since the action of the light would tend to prevent this."

The diffusion of bromine, even in atomic form through the silver bromide crystal lattice is very improbable. It is well known that the diffusion of hydrogen peroxide and ozone does not occur even through very thin layers of mica. In quartz crystals, inclusions of liquid and gaseous carbon dioxide are sometimes found, which, through geological ages were unable to diffuse even through thin layers of the crystal lattice.

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[76] *Loc. cit.*

R. B. Wilsey [77] and W. P. Davey [78] have found the crystal lattice of silver bromide to be simple cubic, the distance between the centers of a silver and of a bromine ion being 2.89 Å.U. Their values agree within one per cent. W. P. Davey [79] estimated the "radius" of the silver ion as 1.18 Å.U. and the "radius" of the bromine ion as 1.73 Å.U. The calculated spherical sizes of the silver and bromine atoms show that the crystal lattice consists of a close packing, which makes the diffusion of bromine without penetration of atoms by other atoms impossible. In an investigation on the diffusion of iodide through silver iodide films, U. R. Evans and L. C. Bannister [80] came to the conclusion "that the iodine passes through pores of a few molecular diameters in loose union with the silver iodide, a process intermediate in character between diffusion in solid solution and gaseous leakage through definite cracks." The silver-iodide being crystalline in structure does not show an amicroscopical homogeneity all through and it is quite possible that the same thing is the case with the silver bromide and the silver bromo-iodide crystal, with its numerous inclusions.

It seems that H. Kieser [81] has realized the difficulty of the explanation of the escape of bromine from the interior of a grain to the surface, when he proposed the following explanation:

The inner photo-electric effect (photo-conductivity) liberates from a bromine ion an electron, forming a bromine atom. The charges of the surrounding silver ions in the lattice are now no more in equilibrium and a site of disturbance with a positive charge is formed. Under the influence of this charge, particularly along the lattice lines of the bromine ions which are nearest to the surface of the crystal, a strong deformation of the ions in the direction of the place of disturbance appears. The bromine ion that forms the end of the lattice lines at the surface is not completely compensated for its charge, so that an equilibrium will have to be established if a

[77] *Phil. Mag.*, **42**: 262 (1921).
[78] *Phys. Rev.*, **19**: 248 (1922).
[79] *Phys. Rev.*, **18**: 102 (1921).
[80] *Proc. Roy. Soc.*, **A125**: 394 (1929).
[81] *Loc. cit.*

displacement of the valency electrons in the lattice lines of the bromine ions takes place. The bromine ion at the surface of the grain then becomes a bromine atom. As far as the mechanism is concerned for the release of electrons it does not differ from the orientation theory of the latent image formation as proposed by S. E. Sheppard, A. P. H. Trivelli and R. P. Loveland,[82] and amplified in Sheppard's Hurter and Driffield lecture.[83] It is difficult to see in which way this picture explains reversal. If the investigations of O. Lehmann,[84] C. Tubandt, P. Eggert and G. Schibbe,[85] A. Benrath and J. W. Wainoff [86] and others have shown that by the electrolysis of solid silver halide no halogen is liberated at the anode, we may reject H. Kieser's picture.

The investigations of U. R. Evans and L. C. Bannister are important for the knowledge of the formation of a latent image within the grain in the exposure regions of maximum density and solarization. Lüppo-Cramer [87] showed that a solarized exposed silver halide plate after treatment with potassium iodide solution develops rapidly to a normal condition without a trace of solarization. Potassium iodide breaks up the grain crystal into smaller particles by forming silver iodide out of silver bromide and exposing developable specks to the developer (*Keimblosslegung*). At the surface of the grain lies a solarized latent image and under the surface, a normal latent image or, expressed in the terms of the concentration speck theory: *at the surface of the grain are smaller specks than within the grain.* This explains why solarization cannot be detected with a physical developer after primary fixation. Only very fine grain emulsions containing particles of the size of small colloidal silver may show solarization in this way because no developable speck can be formed within a grain that is too small for it. Fine grain emulsions with microscopically visible grains often contain many grains of submicroscopical and amicroscopical dimensions, which give

[82] *Loc. cit.*
[83] *Phot. J.*, **48**: 397 (1928).
[84] *Ann. Physik*, **24**: 1 (1885).
[85] *Z. anorg. Chem.*, **110**: 196 (1920); **115**: 105 (1921); **117**: 1 and 48 (1921).
[86] *Z. physik. Chem.*, **77**: 257 (1911).
[87] *Eder's Handb.*, **II, 1**: 531 (1927).

solarization after primary fixation. Therefore, the appearance of solarization after primary fixation is not only a question of exposure, but also of grain size and range of grain size distribution. This explains the many contradictory statements in the literature of J. J. Sterry,[88] F. Kogelmann,[89] H. Weiss,[90] J. M. Eder,[91] B. Homolka,[92] and Lüppo-Cramer.[93]

This explanation seems to contradict the criticism of T. Slater-Price. Even if we admit that there is no relation between the speck at the surface of the grain and the specks within the grain, because of amicroscopical heterogeneity it remains a problem why there should be a higher halogen pressure or halogen concentration at the surface of the grain than within the grain, so out of proportion that in spite of the stronger light intensity the specks at the surface are smaller than within the grain. The concentration speck theory is able to solve qualitatively another problem of the solarization. F. W. T. Krohn,[94] R. B. Wilsey, H. A. Pritchard, and the author [95] have found that the solarization occurs only with partial development and disappears with complete development. The heavy exposures reduced the rate of development, but did not diminish the density to be obtained if development was sufficiently prolonged. As mentioned above, in order to introduce developability the sensitivity speck must reach a certain size with exposure. This size is not so sharply fixed that above this size there is only developability and below this size only undevelopability. There is a region of gradual change; that means at a certain minimum size of developability it takes longer time to introduce developability if the speck is smaller. We regard the first reversal of the solarization as a diminution of the size of a developable center. It is understandable that this varies with different grains that have different sizes of specks and results in the described development effect. Solarization, therefore, is not to be

[88] *Eder's Jahrb.*, 289 (1899).
[89] *Loc. cit.*
[90] *Loc. cit.*
[91] *Photochemie*, 293 (1906).
[92] *Phot. Korr.*, 168 (1907).
[93] *Phot. Korr.*, 260 (1905).
[94] *Phot. J.*, **64**: 369 (1924).
[95] *J. Opt. Soc. Amer.*, **12**: 661 (1926).

regarded as a removal of the development speck but as a small diminution of size, by which a number of these specks (not all) fall below the developability for shorter development time.

Summarizing, we may say that the concentration speck theory in combination with the regression theory of the solarization is qualitatively a satisfactory theory to explain all the known phenomena. It is in agreement with the correlations found by the author and E. C. Jensen [96] between the normal exposure region and the solarization region in their investigation of correlations, if one can picture the source of the increased halogen concentration during the overexposure. Lüppo-Cramer is correct in his statement that in spite of the fact that every effort to picture the formation of the solarized latent image has failed, this is no reason to reject the regression theory. There is too much evidence in favor of it.

A TENTATIVE HYPOTHESIS OF THE FORMATION OF THE SOLARIZED LATENT IMAGE ON THE BASIS OF THE REGRESSION THEORY.

In the formation of the normal latent image we had to solve the problem of the movement of the silver ion. In the formation of the solarized latent image the problem of movement of the bromine atom must be solved.

We must abandon the idea that the bromine is coming from the interior of the grain to diminish the size of the speck at the surface of the grain for visible light exposures. That means *the process of the formation of the solarized latent image for visible light exposures is mainly restricted to the surface of the grain.* This is quite possible because of the fact that halogen vapor is able to diffuse through dry gelatin in thin layers. If bromine is liberated on the surface of the silver bromo-iodide grain this will react with the iodide of the crystal lattice and change places. The reactions then will take place between silver and iodine. The larger grains containing more silver iodide than the smaller ones may show more silver $\rightleftarrows$ iodine reaction; the smaller grains containing less silver iodide than the larger ones may show more silver $\rightleftarrows$ bromine reaction. This depends on the distribution of the iodine ions in the grain.

[96] *Loc. cit.*

K. C. D. Hickman [97] obtained evidence that while in bulk bromine reacts with silver sulfide to give silver bromide, sulfuric and hydrobromic acids, on the other hand, when formed in limited quantity by the photochemical decomposition of silver bromide grains, as in a photographic exposure, bromine then reacts with the silver sulfide of the sensitivity speck in such a way as to give rise to metallic silver instead of silver bromide, and possibly in greater proportion than one silver atom for each silver halide pair decomposed. Silver is supposed to be more effective than its equivalent of silver sulfide in producing developability. It may be said that this hypothesis has been shown to be thermodynamically feasible by R. H. Lambert and E. P. Wightman.[98] Since the inception of the *Acceptor Theory*, K. C. D. Hickman (in a personal communication) has had this view of the course of exposure and over-exposure that the first action of light is to liberate a *little* halogen, which will attack the silver sulfide speck, generating metallic silver; and that the continued action is to liberate an excess which will then, or probably *afterwards*, rebrominate the silver with consequent loss of developing power. It is a good point that K. C. D. Hickman called attention to the fact that the rebromination ought to increase in the dark period between exposure and development.

In Hickman's theory, as in all other theories, there are two reactions—the formation of silver, and the rebromination of silver. Both reactions run in opposite directions. The amount of silver sulfide being limited will sooner or later be exhausted as an acceptor. Therefore, the total amount of silver formed will first increase until a maximum is reached and then will decrease. The decrease is kept in equilibrium with the decomposition of the surrounding silver halide through the halogen pressure surrounding the speck.

According to the electrolytic theory of the concentration speck theory the silver ions liberated in the surrounding silver halide are collected around the speck and therefore the total amount of silver of the speck will increase as long as the halogen, which gives the regressive action, comes from the same silver halide that gives the equivalent amount of silver

[97] *Phot. J.*, 67: 34 (1927).
[98] *J. Phys. Chem.*, 31: 1249 (1927).

to the speck. Only if more bromine is liberated far enough from the speck and less silver ions (or none) are collected on the speck, it is possible that the total number of silver atoms of the speck goes over a maximum. This means that the maximum of Hickman's theory is extended to greater exposures in the concentration speck theory. The appearance of a maximum of silver in the speck depends, therefore, on the exhaustion of the concentration action of the speck. This has been derived previously from observations of the author with R. P. Loveland and E. C. Jensen [99] and it was found that the exhaustion takes place around the maximum density, depending on the size of the silver sulfide specks. The smaller specks are sooner exhausted than the larger ones.

The action of the photochemically liberated halogen on the concentration speck is to produce an increase of the size of the speck until a maximum size is reached, after which it diminishes.

After the exhaustion of the concentration action of the speck it acts further only as a spot of disturbance in the crystal lattice, which grows slowly during continued exposures, when at the same time the photochemically liberated halogen diffuses a way through the gelatin. This means in the solarization region two reactions remain running in opposite directions; one diminishing the size of the speck through increasing pressure of the halogen, which is no longer absorbed by silver sulphide and therefore dominates, and the other increasing the size of the speck by the printing-out decomposition. This means that the size of the speck will go through a minimum again, after which the second reversal of the solarization appears.

Solarization is present in collodion plates and on daguerreotype plates, and no silver sulfide specks are there. Any halogen absorbing speck, other than silver sulfide, may take its place—even pure silver. The amicroscopical heterogeneity of the silver grain surface and the diffusion of halogen to compensate for its pressure, will give on the speck maxima and minima, which explain the solarization.

For X-rays a similar explanation holds. We have to regard the formation of the X-ray latent image as quite

[99] *Loc. cit.*

different from that of light as is pointed out by S. E. Sheppard and the author,[100] and by L. Silberstein and the author[101] in a new investigation of the quantum theory of X-ray exposures on pure silver bromide grains. The X-ray quantum, having about 1,000 to 5,000 times more energy than the blue-violet light quantum, liberates, on the spot of absorption, an electron which has such speed that it is able to liberate 1000 to 2000 silver atoms from the neighboring silver halide crystal lattice. These silver atoms form a center of developability. For visible-light exposures, these centers are restricted to the surface of the grain. For X-ray exposures they are distributed through the whole grain. The presence of centers of developability at the surface of the grain determine the beginning of development. For X-ray exposures there is a much greater probability that centers of developability will be formed within the grain than on the surface of the grain. We may, therefore, expect that around every center at the surface of the grain in the overexposure region of solarization there are distributed a number of developable centers, which will not start development of the whole grain because the developer is not able to reach them. The liberated halogen from these centers will move towards the surface through cracks or break through thin walls of the lattice which separates them from centers at the surface of the grain. The result will be a diminution in size of the specks at the surface, resulting in a diminution of developability.

This is an explanation of the first reversal of the solarization similar to that of Lüppo-Cramer for visible light. This theory is applicable to X-rays because T. Slater-Price's criticism holds for visible exposures but not for X-ray exposures.

SUMMARY.

I. A general discussion is given of the main theories proposed to explain solarization as a development and as a latent image problem.

II. A critical investigation leads to the rejection of the progression theory of coagulation of the latent image silver

[100] *Phot. J.*, **66**: 505 (1926).

[101] To be published.

by prolonged exposures and the acceptance of the regression theory.

III. The regression theory is applied to the concentration speck theory, which leads to a tentative hypothesis of the solarized latent image.

IV. Lüppo-Cramer's explanation of solarization holds for X-rays.

The Reflective Power of Some Substances in the Ultraviolet. P. R. GLEASON. (*Proc. Nat. Acad. Sciences*, July, 1929.) For the range of wave-lengths from 585 Å. to 1850 Å. there have been no data on reflecting powers for individual frequencies. In this investigation light from discharge tubes containing, hydrogen, helium, oxygen or nitrogen was analyzed in a vacuum spectrograph by reflection from a concave speculum grating. Light of a particular wave-length was made to fall on a photographic plate either directly or after reflection from the substance whose reflecting power was to be measured. "In practice the exposure times of the direct and reflected spectra were adjusted so as to give the same opacity of the spectral line image in both cases. If the source remained constant in intensity throughout, the reflecting power was given by the ratio of the two exposure times." The following substances were worked with,—aluminium, beryllium, brass, chromium, gold, nickel, platinum, silicon, silver, speculum steel, stellite, fluorite, glass and quartz For the wave-length 585 Å. freshly sputtered platinum has the largest reflecting power, 18.6 per cent., and this it keeps for many months. In its fresh state its reflecting power is more than double that of any common substance used for gratings. "Nickel, crystalline quartz and gold are the only other substances reflecting more than 10 per cent. at 585 Å. Chromium and silicon are both superior at the longer wave-lengths but do not hold up to a high value at the shorter wave-lengths." Quartz, glass and fluorite have a larger reflecting power near 1200 Å. than at 585 or 1850 Å. Beryllium for 1640 Å. shows a reflecting power of only 1 per cent. This may mean that its index of refraction for this wave-length is unity.

G. F. S.

DYNAMICAL ANALYSIS OF MACHINES.*

BY

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FOUR BAR LINKAGE MECHANISM.

WE will now consider the kinematics of the four bar linkage and determine the velocity ratios, their derivatives and the acceleration in terms of the fundamental coördinate: (1) by an anlytical procedure, (2) by the use of velocity and acceleration diagrams, and (3) by the method of centers.

(1) Analytical Procedure (Fig. 19):

The number "d" will be assumed fixed; i.e., the frame. Then the condition equations are:

$$a \cos \theta + b \cos \psi + c \cos \phi = d,$$
$$a \sin \theta + b \sin \psi - c \sin \phi = 0.$$

Differentiating we obtain the velocity relations:

$$-a\dot{\theta} \sin \theta - b\dot{\psi} \sin \psi - c\dot{\phi} \sin \phi = 0,$$
$$a\dot{\theta} \cos \theta + b\dot{\psi} \cos \psi - c\dot{\phi} \cos \phi = 0.$$

From these we can readily obtain the angular velocity ratios:

$$K_{\psi\theta} = \frac{\dot{\psi}}{\dot{\theta}} = -\frac{a \sin (\theta + \phi)}{b \sin (\phi + \psi)} = \frac{d\psi}{d\theta}$$

and

$$K_{\phi\theta} = \frac{\dot{\phi}}{\dot{\theta}} = -\frac{a \sin (\theta - \psi)}{c \sin (\psi + \phi)} = \frac{d\phi}{d\theta}.$$

If Z_a, Z_b and Z_d are the respective distances to the c. of g. of members a, b and c, from hinge joints A, B and D, the linear velocity ratios of the c. of g. of the members with respect to the fundamental coördinate θ, are:

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* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

Continued from p. 258, vol. 209, February, 1930.

$$K_{a\theta} = \frac{\dot{S}_a}{\dot{\theta}} = Z_a \qquad \text{and} \qquad K_{c\theta} = \frac{\dot{S}_b}{\dot{\theta}} = \frac{Z_c\dot{\phi}}{\dot{\theta}} = Z_c K_{\phi\theta}$$

and for the c. of g. of the member b, since:

$$x_b = a\cos\theta + Z_b\cos\psi,$$
$$y_b = a\sin\theta + Z_b\sin\psi,$$

$$K_{bx\theta} = \frac{\dot{x}_b}{\dot{\theta}} = -a\sin\theta - Z_b K_{\psi\theta}\sin\psi,$$

$$K_{by\theta} = \frac{\dot{y}_b}{\dot{\theta}} = a\cos\theta + Z_b K_{\psi\theta}\cos\psi.$$

For the derivatives of the velocity ratios:

$$\frac{dK_{\psi\theta}}{d\theta} = -\frac{a}{b}\frac{\cos(\theta+\phi)}{\sin(\phi+\psi)}(1+K_{\phi\theta}) + \frac{a}{b}\frac{\sin(\theta+\phi)}{\sin^2(\phi+\psi)}\cos(\phi+\psi)(K_{\phi\theta}+K_{\psi\theta})$$

$$= -\frac{a}{b}\frac{\cos(\theta+\phi)}{\sin(\phi+\psi)} + \frac{a}{b}\frac{\sin(\theta+\phi)\cos(\phi+\psi)}{\sin^2(\phi+\psi)}K_{\psi\theta} + \frac{a}{b}\frac{\sin(\theta-\psi)}{\sin^2(\phi+\psi)}K_{\phi\theta}$$

$$= -\frac{a}{b\sin^3(\phi+\psi)}\left[\sin^2(\phi+\psi)\cos(\theta+\phi) + \frac{a}{b}\sin^2(\theta+\phi)\cos(\phi+\psi) + \frac{a}{c}\sin^2(\theta-\psi)\right],$$

$$\frac{dK_{\phi\theta}}{d\theta} = -\frac{a}{c}\frac{\cos(\theta-\psi)}{\sin(\psi+\phi)}(1-K_{\psi\theta}) + \frac{a}{c}\frac{\sin(\theta-\psi)}{\sin^2(\psi+\phi)} \times \cos(\psi+\phi)(K_{\psi\theta}+K_{\phi\theta})$$

$$= -\frac{a}{c}\frac{\cos(\theta-\psi)}{\sin(\psi-\phi)} + \frac{a}{c}\frac{\sin(\theta-\psi)\cos(\phi+\psi)}{\sin^2(\psi+\phi)}K_{\phi\theta} + \frac{a}{c}\frac{\sin(\theta+\phi)}{\sin^2(\psi+\phi)}K_{\psi\theta}$$

$$= \frac{a}{c\sin^3(\psi+\phi)}\left[\sin^2(\psi+\phi)\cos(\theta-\psi) + \frac{a}{c}\sin^2(\theta-\psi)\cos(\phi+\psi) + \frac{a}{b}\sin^2(\theta+\phi)\right].$$

From these we may readily calculate:

$$\frac{dK_{a\theta}}{d\theta} = 0 \qquad \text{and} \qquad \frac{dK_{c\theta}}{d\theta} = Z_c \frac{dK_{\phi\theta}}{d\theta},$$

$$\frac{dK_{bx\theta}}{d\theta} = -a\cos\theta - Z_b K_{\psi\theta}^2 \cos\psi - Z_b \frac{dK_{\psi\theta}}{d\theta}\sin\psi,$$

$$\frac{dK_{by\theta}}{d\theta} = -a\sin\theta - Z_b K_\theta^2 \sin\psi + Z_b \frac{dK_{\psi\theta}}{d\theta}\cos\psi.$$

By the method of Velocity and Acceleration Diagrams Fig. 20:

For the motion of b relative to d, we have a rotation about the instantaneous center I_{bd}:

$$\frac{v_c}{v_b} = \frac{I_{bd}C}{I_{bd}B} = \frac{AN}{AB}, \qquad \text{where } AN \| DC,$$

but

$$\omega_{ad} = \frac{v_b}{a}; \qquad \omega_{cd} = -\frac{v_c}{c}, \qquad \left|\begin{matrix}\omega_{ad} = \dot{\theta} \\ \omega_{cd} = \dot{\phi}\end{matrix}\right|$$

where

$$\frac{w_{cd}}{w_{ad}} = -\frac{v_c}{v_b}\frac{a}{c} = -\frac{AN}{a}\frac{a}{c} = -\frac{AN}{c};$$

$$\therefore \quad \overline{K}_{ca} = \frac{\dot{\phi}}{\dot{\theta}} = -\frac{AN}{c} = -\frac{I_{ac}A}{I_{ac}D} = -\frac{AN'}{DC'},$$

but

$$AN' = a\sin(\theta - \psi); \qquad DC' = c\sin(\psi + \phi);$$

$$\therefore \quad K_{\phi\theta} = -\frac{a}{c}\frac{\sin(\theta - \psi)}{\sin(\psi + \phi)}.$$

By a direct velocity resolution along b:

$$v_b \sin(\theta - \psi) = v_c \sin(\psi + \phi),$$

which on substituting for $v_b = a\dot{\theta}$ and $v_c = -c\dot{\phi}$, we obtain the previous velocity ratio.

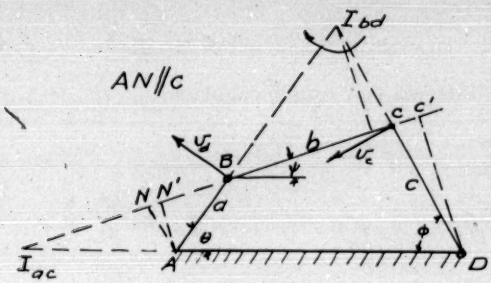

FIG. 20

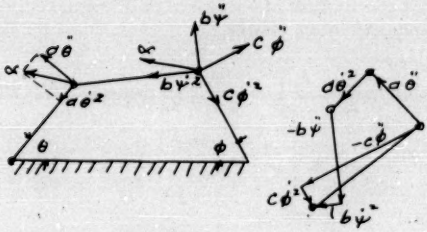

FIG 21

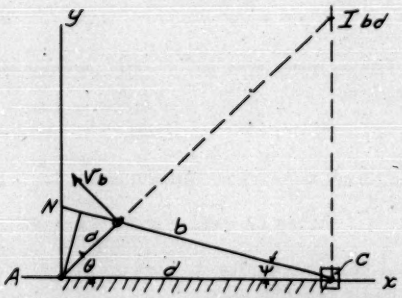

FIG 22

To obtain the velocity ratio $K_{\psi\theta}$, we note:

$$\dot{\psi} = \frac{\bar{v}_c - \bar{v}_b}{b} = -\frac{v_b \cos(\theta - \psi) + v_c \cos(\psi + \phi)}{b},$$

$$\frac{\dot{\psi}}{\dot{\theta}} = -\left[\frac{a}{b}\cos(\theta - \psi) - \frac{c}{b} K_{\phi\theta} \cos(\psi + \phi)\right];$$

$$\therefore \quad K_{\psi\theta} = -\frac{a}{b}\frac{\sin(\phi + \theta)}{\sin(\psi + \phi)}.$$

By the method of three centers, we have:

$$\frac{\dot{\phi}}{\dot{\theta}} = \frac{\omega_{cd}}{\omega_{ad}} = -\frac{I_{ac}A}{I_{ac}D} = K_{\phi\theta}$$

and

$$\frac{\dot{\psi}}{\dot{\theta}} = \frac{\omega_{bd}}{\omega_{ad}} = -\frac{a}{I_{bd}B} = \overline{K}_{\phi\theta},$$

where

$$\frac{I_{ac}A}{I_{ac}D} = \frac{AN'}{DC'} = \frac{a \sin(\theta - \psi)}{c \sin(\psi + \phi)}$$

and

$$\frac{a}{I_{bd}B} = \frac{NB}{b} = \frac{a}{b}[\cos(\theta - \psi) + \sin(\theta - \psi)\cot(\phi + \psi)]$$

$$= \frac{a \sin(\phi + \theta)}{b \sin(\phi + \psi)}.$$

For the acceleration diagram (Fig. 21):

$$-a\ddot{\theta}\sin\theta - b\ddot{\psi}\sin\phi - a\dot{\theta}^2\cos\theta - b\dot{\psi}^2\cos\psi = c\ddot{\theta}\sin\phi + c\dot{\phi}^2\cos\phi,$$

$$a\ddot{\theta}\cos\theta + b\ddot{\psi}\cos\psi - a\dot{\theta}^2\sin\theta - b\dot{\psi}^2\sin\psi = c\ddot{\phi}\cos\phi - c\dot{\phi}^2\sin\phi,$$

which thus reduce to the previous analytical expression for the acceleration.

SLIDING BLOCK LINKAGE MECHANISM.

For the motion of b relative to d, we have a rotation about the instantaneous center I_{bd} (Fig. 22):

$$\frac{v_c}{v_b} = \frac{I_{bd}C}{I_{bd}B} = \frac{AN}{AB} = \frac{AN}{a},$$

but

$$\dot{\theta} = \omega_{ad} = \frac{v_b}{a}; \qquad \frac{v_c}{\omega_{ad}} = -\frac{\dot{x}}{\dot{\theta}} = AN = -K_{x\theta}$$

hence

$$K_{x\theta} = -\frac{a \sin(\theta + \psi)}{\cos \psi} = \frac{\dot{x}}{\dot{\theta}} = -\frac{v_c}{\dot{\theta}},$$

which is immediately verified by a direct resolution, along the connecting rod b:

$$a\dot{\theta} \sin(\theta + \psi) = -\dot{x} \cos \psi.$$

For the angular velocity ratio $K_{x\theta}$, we have, by equating the common tangential velocity at B, or by the method of three centers:

$$\frac{\dot{\psi}}{\dot{\theta}} = \frac{\omega_{bd}}{\omega_{ad}} = \frac{a}{I_{bd}B} = \frac{BN}{b} = K_{\psi\theta}.$$

This ratio may be obtained from the vector difference of the velocities at B and C, since

$$\dot{\psi} = \frac{\bar{v}_b - \bar{v}_c}{b} = \frac{v_b \cos(\psi + \theta) + v_c \sin \psi}{b};$$

$$\therefore \quad \frac{\dot{\psi}}{\dot{\theta}} = \frac{1}{b}(a \cos(\psi + \theta) - K_{x\theta} \sin \psi) = K_{\psi\theta};$$

$$\therefore \quad K_{\psi\theta} = \frac{a}{b}\frac{\cos \theta}{\cos \psi} = \frac{a \cos \theta}{\sqrt{b^2 - a^2 \sin^2 \theta}}.$$

To calculate the space rate of changes of the velocity relative to the primary coördinate θ, we have:

$$\frac{dK_{x\theta}}{d\theta} = -\frac{a \cos(\theta + \psi)}{\cos \psi}(1 + K_{\psi\theta}) - \frac{a \sin(\theta + \psi)}{\cos^2 \psi} \sin \psi \cdot K_{\psi\theta}$$

$$= -\frac{a}{\cos \psi}\left[\cos(\theta + \psi) + \frac{a}{b}\frac{\cos^2 \theta}{\cos^2 \psi}\right]$$

and

$$\frac{dK_{\psi\theta}}{d\theta} = -\frac{a}{b}\frac{\sin \theta}{\cos \psi} + \frac{a}{b}\frac{\cos \theta}{\cos^2 \psi} \cdot \sin \psi K_{\psi\theta}$$

$$= -\frac{a}{b \cos \psi}\left(\sin \theta + \frac{a}{b}\frac{\cos^2 \theta}{\cos^2 \psi} \sin \psi\right).$$

For the velocity ratios and their derivatives for the c. of g. of the connecting rod b, we may consider the motion divided into x and y components. Then, if z_0 is the distance of the c. of g. from crosshead C, we have:

$$\dot{x}_b = -\dot{x} - z_0\dot{\psi}\sin\psi = -a\dot{\theta}\sin\theta - (b - z_0)\dot{\psi}\sin\psi,$$
$$\dot{y}_b = z_0\dot{\psi}\cos\psi = a\dot{\theta}\cos\theta - (b - z_0)\dot{\psi}\cos\psi,$$

so that

$$K_{xb\theta} = -K_{x\theta} + z_0K_{\psi\theta}\sin\psi = -a\sin\theta - (b - z_0)K_{\psi\theta}\sin\psi,$$
$$K_{yb\theta} = z_0K_{\psi\theta}\cos\psi = -a\cos\theta - (b - z_0)K_{\psi\theta}\cos\psi,$$

hence

$$K_{xb\theta} = -\frac{a\sin(\theta + \psi)}{\cos\psi} + Z_0\frac{a}{b}\cos\theta\tan\psi,$$
$$K_{yb\theta} = Z_0\frac{a}{b}\cos\theta.$$

The derivatives of these velocity ratios are:

$$\frac{dK_{xb\theta}}{d\theta} = -\frac{a\cos(\theta + \psi)}{\cos\psi}(1 + K_{\psi\theta}) - \frac{a\sin(\theta + \psi)}{\cos^2\psi}\sin\psi$$
$$- z_0\frac{a}{b}(\sin\theta\tan\psi - \cos\theta\sec^2\psi)$$
$$\frac{dK_{yb\theta}}{d\theta} = -z_0\frac{a}{b}\sin\theta.$$

For the acceleration diagram of the sliding block linkage: Equating the common acceleration at A, we have, Fig. 23:

$$-a\ddot{\theta}\sin\theta - a\dot{\theta}^2\cos\theta = \ddot{x} + b\ddot{\psi}\sin\psi + b\dot{\psi}^2\cos\psi,$$
$$a\ddot{\theta}\cos\theta - a\dot{\theta}^2\sin\theta = b\ddot{\psi}\cos\psi - b\dot{\psi}^2\sin\psi,$$

so that on combining:

$$\ddot{x} = -\frac{a\ddot{\theta}\sin(\theta + \psi)}{\cos\psi} - \frac{(a\cos(\theta + \psi) + bK_{x\theta}^2)}{\cos\psi}\dot{\theta}^2$$
$$= \frac{a\cos\theta}{b\cos\psi}\ddot{\theta} - \frac{(a\sin\theta - bK_{\psi\theta}^2\sin\psi)}{b\cos\psi}\dot{\theta}^2,$$

where

$$K_{\psi\theta} = \frac{a\cos\theta}{b\cos\psi}.$$

SWINGING BLOCK LINKAGE MECHANISM ROTARY ENGINE.

The motion of the block is compounded of the motion of the frame $x\dot{\theta}$ and the relative velocity $-\dot{x}$. For the velocity ratios with respect to the coördinate θ of the rotating frame, we have the kinematic condition (Fig. 24)

$$l\dot{\psi}\cos(\psi-\theta)=x\dot{\theta} \qquad \text{and} \qquad l\dot{\psi}\sin(\psi-\theta)=-\dot{x},$$

so that

$$K_{\psi\theta}=\frac{\dot{\psi}}{\dot{\theta}}=\frac{x}{l}\sec.(\psi-\theta); \qquad K_{x\theta}=\frac{\dot{x}}{\dot{\theta}}=-x\tan(\psi-\theta)$$

and

$$\frac{dK_{\psi\theta}}{d\theta}=\frac{x}{l}\tan(\psi-\theta)\sec.(\psi-\theta)(K_{\psi\theta}-2)$$

$$=\frac{x}{l}\tan(\psi-\theta)\sec.(\psi-\theta)\left(\frac{x}{l}\sec.(\psi-\theta)-2\right),$$

$$\frac{dK_{x\theta}}{d\theta}=-K_{x\theta}\tan(\psi-\theta)-x\sec^2(\psi-\theta)(K_{\psi\theta}-1)$$

$$=x\tan^2(\psi-\theta)-\frac{x}{l}\sec^2(\psi-\theta)\cdot[x-l\cos(\psi-\theta)].$$

For the acceleration diagram we have the conditions shown in Fig. 25.

The acceleration of the sliding block consists, of the relative acceleration $-\ddot{x}$, the Cariolis component $-2\dot{x}\dot{\theta}$ the centripetal frame component $x\dot{\theta}^2$ and the tangential acceleration of the frame $x\ddot{\theta}$. The pin in the block also can be expressed in terms of the motion of the extremity of the connecting rod, so that we have on equating:

$$-\ddot{x}+x\dot{\theta}^2=l\ddot{x}\sin(\psi-\theta)+l\dot{\psi}^2\cos(\psi-\theta)$$
$$x\ddot{\theta}+2\dot{x}\dot{\theta}=l\ddot{\psi}\cos(\psi-\theta)-l\dot{\psi}^2\sin(\psi-\theta).$$

Combining,

$$\ddot{\psi}=\frac{x}{l}\sec(\psi-\theta)\cdot\ddot{\theta}+\frac{x}{l}\tan(\psi-\theta)\sec(\psi-\theta)$$
$$\times\left(\frac{x}{l}\sec(\psi-\theta)-2\right)\dot{\theta}^2$$

and

$$\ddot{x} = -x\tan(\psi-\theta)\ddot{\theta} + x\tan^2(\psi-\theta)\dot{\theta}^2 - x\sec^2(\psi-\theta) \times\left(\frac{x}{l}\sec^2(\psi-\theta) - 1\right)\dot{\theta}^2.$$

These equations can be verified by a direct differentiation of the velocity ratios, thus:

$$\frac{\dot{x}}{\dot{\theta}} = -x\tan(\psi-\theta) \quad \text{and} \quad \frac{\dot{\psi}}{\dot{\theta}} = \frac{x}{l}\sec(\psi-\theta).$$

Therefore,

$$\begin{aligned}\ddot{x} &= -x\tan(\psi-\theta)\ddot{\theta} - \dot{x}\tan(\psi-\theta)\dot{\theta} \\ &\qquad - x\dot{\theta}\sec^2(\psi-\theta)\cdot(\dot{\psi}-\dot{\theta}) \\ &= -x\tan(\psi-\theta)\ddot{\theta} + x\tan^2(\psi-\theta)\dot{\theta}^2 \\ &\qquad - x\sec^2(\psi-\theta)\left(\frac{\dot{\psi}}{\dot{\theta}} - 1\right)\dot{\theta}^2 \\ &= -x\tan(\psi-\theta)\ddot{\theta} + x\tan^2(\psi-\theta)\dot{\theta}^2 \\ &\qquad - x\sec^2(\psi-\theta)\left(\frac{x}{l}\sec(\psi-\theta) - 1\right)\dot{\theta}^2\end{aligned}$$

and

$$\begin{aligned}\ddot{\psi} &= \frac{x}{l}\sec(\psi-\theta)\ddot{\theta} + \frac{\dot{x}}{l}\sec(\psi-\theta)\dot{\theta} + \frac{x}{l}\sec(\psi-\theta) \\ &\qquad \times\tan(\psi-\theta)\left(\frac{\dot{\psi}}{\dot{\theta}} - 1\right)\dot{\theta} \\ &= \frac{x}{l}\sec(\psi-\theta)\ddot{\theta} + \frac{x}{l}\tan(\psi-\theta)\sec(\psi-\theta) \\ &\qquad \times\left(\frac{x}{l}\sec(\psi-\theta) - 2\right)\dot{\theta}^2.\end{aligned}$$

DYNAMIC PROPERTIES OF MEMBERS.

The kinetic energy of a member is given by the expression:

$$2T = M(v^2 + k_g^2\phi^2) = M(\dot{x}_g^2 + \dot{y}_g^2 + k_g^2\phi^2),$$

where v = the velocity of the c. of g. divided into components $\dot{x}_g$, $\dot{y}_g$ and ϕ is the angular velocity.

Thus if the motion of one joint and the angular velocity are known:

Then, Fig. 26,

$$2T = M[v_A^2 \cos\alpha + (z_g\dot{\phi} - v_A \sin\alpha)^2 + k_g^2\dot{\phi}^2]$$
$$= M[v_A^2 - 2z_g v_A\dot{\phi}\sin\alpha + k_A^2\dot{\phi}^2],$$

where

$$k_A^2 = k_g^2 + Z_g^2.$$

When a member translates in a rotating frame, as in the connecting rod of an oscillating engine (Fig. 27):

$$2T = M[(Z_c - Z_g)^2\dot{\phi}^2 + \dot{Z}_r^2 + k_g^2\dot{\phi}^2],$$

Z_r = relative velocity . . ., where

$$Z_r = r\dot{\theta}\sin(\theta + \phi), \quad \dot{\phi} = \frac{r}{Z_c}\dot{\theta}\cos(\theta + \phi), \quad r\dot{\theta} = v_B;$$

$$\therefore \quad 2T = M\left[\left((Z_c^2 - 2Z_cZ_g)\frac{r^2}{Z_c^2}\cos^2(\theta + \phi) + r^2\sin^2(\theta + \phi)\right)\dot{\theta}^2 + k_B^2\dot{\phi}^2\right]$$
$$= M[r^2\dot{\theta}^2 - 2Z_g r\dot{\theta}\cos(\theta + \phi)\dot{\phi} + k_B^2\dot{\phi}^2]$$
$$= M[v_B^2 - 2Z_g v_B\dot{\phi}\cos(\theta + \phi) + k_B^2\dot{\phi}^2],$$

where we note as before $v_B\cos(\theta + \phi)$ is the normal component of the velocity of joint B with respect to the rod.

For the kinetic energy of a member of length l in terms of the end points A and B, we have:

$$\dot{x}_g = \frac{l - Z}{l}\dot{x}_A + \frac{Z}{l}\dot{x}_B,$$

$$\dot{y}_g = \frac{l - Z}{l}\dot{x}_B + \frac{Z}{l}\dot{y}_B,$$

where Z is the distance from A to the c. of g.

Substituting in the expression for T:

$$2T = M\left[(\dot{x}_A^2 + \dot{y}_B^2)\left(\frac{l - Z}{l}\right)^2 + (\dot{x}_B^2 + \dot{y}_B^2)\left(\frac{Z}{l}\right)^2 + 2(\dot{x}_A\dot{x}_B + \dot{y}_A\dot{y}_B)\frac{z(l - Z)}{l^2} + k_g^2\dot{\phi}^2\right]$$

and since

$$\dot{x}_B - \dot{x}_A = -l\dot{\phi}\sin\phi \quad \text{and} \quad \dot{y}_B - \dot{y}_A = l\dot{\phi}\cos\phi$$

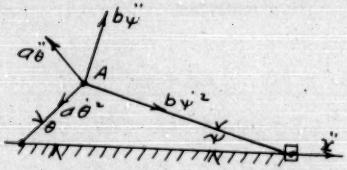

FIG. 23

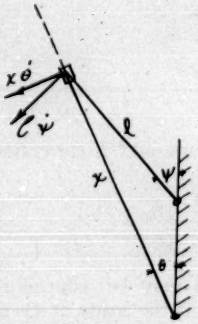

FIG. 24

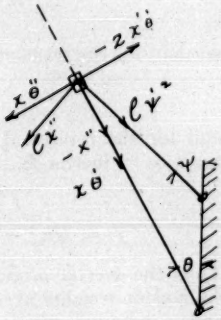

FIG. 25

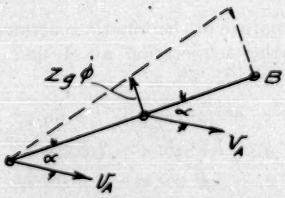

FIG. 26

and

$$\dot{x}_A^2 + \dot{y}_A^2 = v_A^2, \qquad \dot{x}_B^2 + \dot{y}_B^2 = v_B^2,$$

$$2T = M\left[\frac{l-Z}{l}v_A^2 + \frac{Z}{l}v_B^2 + (k_g^2 - Z(l-Z))\dot{\phi}^2\right]$$

$$= \mu_A v_A^2 + \mu_B v_B^2 + M(k_g^2 - Z(l-Z))\dot{\phi}^2$$

$$= \mu_A v_A^2 + \mu_B v_B^2 + M(k_A^2 - Zl)\dot{\phi}^2.$$

INERTIA RESOLUTIONS.

We might have anticipated the above from a simple inertia resolution of the member. If l_1, l_2 are the distances from the end points A and B respectively to the c. of g. of the rod, then,

$$\mu_A = M\frac{l_2}{l}, \qquad \mu_B = M\frac{l_1}{l},$$

so that, for the translation property,

$$\mu_A + \mu_B = M$$

and for the rotational property we must add a correction moment of inertia I_c, such that:

$$I + \mu_A l_1^2 + \mu_B l_2^2 = Mk_g^2;$$

$$\therefore \quad I_c = Mk_g^2 - (\mu_A l_1^2 + \mu_B l_2^2) = M(k_g^2 - l_1 l_2).$$

Thus the inertia properties of the rod may be reduced to equivalent weights at A and B equal to the loads at these ends, provided we introduce a "correction moment of inertia" I_c, so that the kinetic energy of the rod is:

$$T = \tfrac{1}{2}\mu_A v_A^2 + \tfrac{1}{2}\mu_B v_B^2 + \tfrac{1}{2}I_c\dot{\phi}^2.$$

To analyze the nature of the kinetic reactions:

We note that the acceleration at A and B have the following components, Fig. 28:

$$\ddot{x}_A = \ddot{x}_g + l_1\dot{\phi}^2\cos\phi + l_1\ddot{\phi}\sin\phi,$$

$$\ddot{y}_A = \ddot{y}_g + l_1\dot{\phi}^2\sin\phi - l_1\ddot{\phi}\cos\phi,$$

$$\ddot{x}_B = \ddot{x}_g - l_2\dot{\phi}^2\cos\phi - l_2\ddot{\phi}\sin\phi,$$

$$\ddot{y}_B = \ddot{y}_g - l_2\dot{\phi}^2\sin\phi + l_2\ddot{\phi}\cos\phi.$$

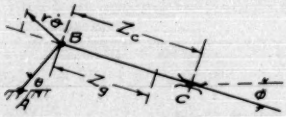

FIG. 27.

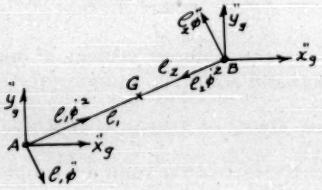

FIG. 28

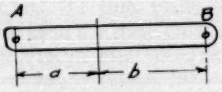

FIG. 29

FIG. 30

FIG. 31

The moment of the kinetic reactions at A and B about G is:

$$(\mu_A l_1^2 + \mu_B l_2^2)\ddot{\phi} = Ml_1l_2\ddot{\phi},$$

since

$$(\mu_A l_1 - \mu_B l_2)\ddot{x}_g \sin\phi = (\mu_A l_1 - \mu_B l_2)\ddot{y}_g \cos\phi = 0.$$

Moreover, this moment is a couple, because

$$\mu_A l_1\ddot{\phi} = \mu_B l_2\ddot{\phi} = M\frac{l_1l_2}{l}\ddot{\phi}.$$

Therefore, to the reduced system of mass points at A and B, we must add a correction moment:

$$Fl = Mk_g^2\ddot{\phi} - Ml_1l_2\ddot{\phi} = M(k_g^2 - l_1l_2)\ddot{\phi},$$

which, since both terms on the right are couples, the correction moment is likewise a couple.

Another resolution is to adjust the position and magnitude of the equivalent mass points, so that (1) the center of gravity, (2) the translation effect and (3) the rotational effect, are equivalent to the actual rod. These are the three specifications that must always be met in any inertia resolution.

TWO MASS ADJUSTABLE DISTANCE METHOD.

The conditions for a dynamical equivalent mass distribution are, Fig. 29:

$$\mu_A + \mu_B = M,$$

$$\mu_A a - \mu_B b = 0,$$

$$\mu_A a^2 + \mu_B b^2 = Mk_g^2.$$

Therefore,

$$\mu_A = \left(\frac{b}{a+b}\right)M = \left(\frac{k_g^2}{a^2 + k_g^2}\right)M,$$

$$\mu_B = \left(\frac{a}{a+b}\right)M = \left(\frac{k_g^2}{b^2 + k_g^2}\right)M,$$

with the condition $ab = k_g^2$. If a or b is defined, then b or a is determined.

THREE OR MORE ADJUSTED MASSES, BUT ARBITRARILY LOCATED.

1. With three masses:

Let μ_0 = the adjusted mass located at the c. of g. μ_1 and μ_2 = the adjusted masses at l_1 and l_2. Then for a dynamical equivalent system (Fig. 30):

$$\mu_1 + \mu_2 + \mu_0 = M,$$
$$\mu_1 l_1 - \mu_2 l_2 = 0,$$
$$\mu_1 l_1^2 + \mu_2 l_2^2 = I_g$$

and

$$\mu_1 l_1 (l_1 + l_2) = \mu_1 l_1 l_2 = \mu_2 l_2 = I_g;$$

$$\therefore \quad \mu_1 = \frac{I_g}{l_1 l}, \qquad \mu_2 = \frac{I_g}{l_2 l},$$

$$\mu_g = M - (\mu_1 + \mu_2) = M - \frac{I_g}{l_1 l_2}.$$

2. With three or more masses:

The conditions for an equivalent system are (Fig. 31):

$$\mu_1 + \mu_2 \cdots \mu_n + \mu_0 = M, \tag{1}$$
$$\mu_1 \bar{r}_1 + \mu_2 \bar{r}_2 \cdots \mu_n \bar{r}_n = 0, \tag{2}$$
$$\mu_1 \bar{r}_1^2 + \mu_2 \bar{r}_2^2 \cdots \mu_n \bar{r}_n^2 = I_g. \tag{3}$$

If we introduce a parameter x and define:

$$a_1 = x n_1 \bar{r}_1 \qquad \text{and} \qquad a_n = x n_n \bar{r}_n,$$

so that on substituting in (3) the parameter is determined; that is,

$$x = \frac{a_1 r_1 + a_2 r_2 \cdots a_n r_n}{I_g}$$

and

$$\mu_1 = \frac{a_1}{r_1 x}, \qquad \mu_2 = \frac{a_2}{r_2 x}, \qquad \mu_n = \frac{a_n}{r_n x},$$
$$\mu_0 = M - (\mu_1 + \mu_2 \cdots + \mu_n).$$

(To be continued)

Measurement of the Velocity of Light by the Use of the Kerr Electro-optical Effect. O. MITTELSTAEDT. (*Ann. der Phys.*, No. 2, 1929.) Fizeau in the nineteenth century measured the velocity of light by sending a beam between two teeth of a revolving wheel to a mirror. From this it was reflected back along the same path. With properly related length of light path and speed of rotation the returning beam was caught upon the cog which had moved up to intercept it while it was going to the mirror and returning to the wheel. From the time and the distance the velocity was calculated. This method is subject to serious difficulties in the measurement of the path of the light, about 40 km., and of the speed of rotation of the wheel which for the success of the observations had to be variable. In 1925 Karolus made use of the Kerr magneto-optical effect instead of a mechanical means of interrupting the beam. Thus increasing the frequency of interruption to 10,000,000 per sec. he reduced the path of the beam to 15 meters. In the present investigation the method is modified and improved. The velocity of light is calculated to be 299,778 km. per sec. Michelson has found 299,796 ± 4 km. per sec.

G. F. S.

Third International Congress for Applied Mechanics will be held at Stockholm, Sweden, August 24 to 29, 1930. The opening meeting will be held at the Royal Academy of Engineering Sciences and regular sessions will be at the Royal Technical University.

The following subjects will be discussed: hydrodynamics; the theory of propulsion and the problem of resistance; elasticity and the strength of materials; oscillations of ships and vehicles, and acoustics of space and similar questions.

A complete program will be announced later. Inquiries concerning the Congress should be sent to the Secretary, Prof. W. Weibull, Kgl. Tekniska Högskolan, Stockholm, Sweden.

R.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

ACCURACY OBTAINABLE WITH PIEZO OSCILLATORS.

In order to secure actual data on their possibilities, as well as to develop and improve piezo oscillators for use as standards of radio frequency, the Bureau of Standards has conducted observations on a number of specially designed piezo oscillators in recent months. The circuit arrangements of all these piezo oscillators are broadly the same, being designed to minimize the effects of external influences on the frequency of operation. The construction is such that the standards are portable and of a size which could be used in a radio transmitting station.

The essential differences among the piezo oscillators were in the design of the quartz plate holder. The following summary indicates the various types of mounting:

600–1 and 200–1; circular plate, thickness vibration, horizontal position with air gap between quartz plate and top electrode, glass spacers, all surfaces plane parallel.

200–3; circular plate, thickness vibration, vertical position with air gap between quartz plate and each electrode, marble spacer, plate held in position rigidly at three points equally spaced around edge. All voltages for operating the piezo oscillator obtained from 110 volt a. c. supply.

N–5; rectangular plate, length vibration, horizontal position with air gap between quartz plate and top electrode, bakelite spacer.

100–1; rectangular plate, length vibration, horizontal position with top plate resting on quartz plate.

25–1; rectangular plate, length vibration, vertical position, electrodes sputtered on quartz plate.

The quartz plates were all kept at their working temperatures continuously except for short accidental interruptions. 100–1 and 200–3 were transported several times by truck to

* Communicated by the Director.

and from the bureau's field station at College Park, Md.; the other piezo oscillators were moved occasionally about the radio laboratory.

The following table gives the results of observation made on these piezo oscillators. Measurements were made each day during the period of observation. The maximum variation gives the worst performance and, in general, the operation was much better. Except in the two cases noted the variations were random and did not show any long-time frequency trend in either a positive or negative direction.

Oscillator No.	Period of Observation.	Average Frequency.	Maximum Variation in Parts per Million.
600–1	8 months	599.913	± 6
200–1	2 "	200.0024	± 3
200–1 [1]	3½ "	200.0014	± 8
200–3	1½ "	200.0013	± 3
N–5 [2]	1½ "	199.9988	± 6
N–5 [3]	1½ "	199.9998	± 5
100–1 [4]	3½ "	99.9998	±15
100–1 [4]	2 "	100.0075	±10
25–1 [5]	3½ "	24.997 [5]	±23

[1] Minor change in circuit arrangement and temperature.

[2] A constant decrease in frequency.

[3] A constant decrease in frequency. Temperature changed.

[4] Change took place at time this piezo oscillator was taken from Bureau of Standards to College Park and returned. Design of plate holder such that changes would be expected when moved.

[5] Initial frequency. Change in frequency was continuous and appears to be approaching a constant value. Change indicates that sputtered electrode is dropping off the plate.

These data indicate that it is possible to construct piezo oscillators for use as frequency standards which are capable of maintaining an accuracy of greater than 1 part per 100,000 over a period of several months. This accuracy satisfies the needs of radio transmitting stations and such standards would be adequate for station standards.

Detailed information concerning the construction of piezo oscillator 200–1 is given in a paper now in press which will appear within a few months and will be announced in these columns.

FOG LANDING DEVELOPMENTS BY THE BUREAU OF STANDARDS.

The bureau is serving as the research division of the aëronautics branch of the Department of Commerce. One of the developments upon which the bureau has been engaged for about a year is the use of radio to aid airplanes landing at an airport in dense fog when the ground is invisible. Numerous methods and devices are being tried, and two more years may be needed to reach a complete solution of the problem.

The pilot landing in fog must be given instantaneous information on his three coördinates in space. That is, he must be guided to and along a suitable runway, must know how far along it he has gone, and the height of the landing path above ground must be properly regulated. Of these three requirements, the first is given by a small directive radio beacon, the second by a marker beacon, and the third by a landing beam. The first of these, the small directive beacon, has been installed by the bureau both at its own field, College Park Md., and at Mitchel Field, L. I., for the Guggenheim Fund. At Mitchel Field, successful blind landings were made by Lt. J. H. Doolittle in September, using this beacon in conjunction with several non-radio instruments.

At College Park all three elements of the system have been installed, the small directive beacon, marker beacons, and the landing beam. The first two are identical in principle with the same devices developed for use on the airways for guiding airplanes from one airport to another; these are described in the bureau's release, "Aircraft radio beacon development by the Bureau of Standards," and in Air Commerce Bulletin, August 15, 1929.

The method now being tried to provide the third element of the system, i.e., to regulate properly the height of the landing path as the airplane approaches the landing, is a directed beam. This beam, of 60,000 to 100,000 kilocycles, is directed at a small angle above the horizontal. A very simple receiving arrangement with visual indicator is used on the airplane. When the airplane is so maneuvered as to keep the deflection of this indicator always at a fixed point, the airplane follows a gliding path down to the ground which

is quite suitable for the landing operation. In one experiment, the landing path marked out by the beam began at approximately three miles from the transmitter; the shape of the path can be judged from the following table:

Distance from Transmitter.	Height Above Ground.
3 miles	2,000 feet
2 "	500 "
1 mile	240 "
½ "	85 "
1,000 feet	15 "
750 "	10 "
500 "	5 "

It will be noted that the slope of this path decreases as the pilot approaches the ground; this tends to facilitate a proper landing.

Each of the three elements of the landing system is being perfected by detailed study and trial flights. Many technical problems remain to be overcome. As the elements of the system are tried in combination, some of them may be entirely altered. The work has advanced to the point, however, where the bureau is convinced that it will be possible to give instantaneous position in three dimensions to a pilot in such manner that he may successfully land in dense fog.

STRENGTH OF WELDED JOINTS IN TUBULAR MEMBERS FOR AIRCRAFT.

THE use of tubular members welded at the joints has become so common in aircraft construction that accurate and reliable data on the strength of welded joints are in great demand. In coöperation with the Aëronautics Branch of the Department of Commerce the Bureau of Standards has made an investigation of the strength of welded aircraft joints. The purpose of this investigation is (1) to assist the Aëronautics Branch in its work of inspecting and licensing planes; (2) to provide fundamental data on the strength of welded aircraft joints for the benefit of the industry.

From replies to questionnaires sent to nineteen leading airplane manufacturers, a test program was prepared which

included determining the strength and efficiencies of some forty welded joints in chromium-molybdenum tubing. These were of three general groups: (1) butt joints tested in tension and compression; (2) T-joints made by welding two tubes together at angles of 90 degrees and tested by supporting one in the testing machine while a load is applied through the other; (3) lattice joints made by welding three tubes together at angles of 60 degrees and tested by loading the middle tube in tension, the other tubes being under compression.

Some joints were made by simply shaping the ends of intersecting tubes to fit around each other, then welding the joints. In others plates, straps, etc. were welded into the joint to determine the additional strength, if any, provided by such reinforcement.

In order that all the welding might be uniform and of good commercial quality a welding procedure control was prepared for this investigation by the American Bureau of Welding.

The efficiencies of butt joints averaged about 88 per cent. in tension and 85 per cent. in compression.

It was found that inserted gusset plates and U-straps were the best methods of reinforcing T and lattice joints. Where tubes are subjected to high bending stresses it is probably more satisfactory to gain strength by increasing the size of the tube than by adding plates, straps, etc. at the joint.

In loading T-joints with the supports as close together as possible a joint having a U-strap was about 30 per cent. stronger than the unreinforced joint. The strength of the lattice joint can be increased about 20 per cent. by either inserting a gusset plate in all tubes at the joint, or welding a U-strap around it.

A joint made with one gusset plate inserted through all tubes is stronger than one with several plates fitted in the angles between the tubes and welded without slotting the tubes.

The report, including drawings of each joint and graphs of the test results, has been completed and will be published soon by the National Advisory Committee for Aëronautics.

ACCELERATED WEATHERING OF PAINTS.

THE item on accelerated exposures of lithopone, lead-zinc, and white lead paints, in Technical News Bulletin No. 152 (December, 1929) has apparently been misinterpreted. The bureau is not recommending certain kinds of white paint and discouraging the use of others. The object of the experiments is to evaluate an accelerated testing procedure. In making this comparison of the accelerated procedure and outdoor exposure, the bureau preferred to make its own paints rather than to use any particular brands. Some paints were also desired in this series which the bureau could be reasonably sure would give good service and at least one that would not prove durable. Knowing that a white paint made of lithopone without admixture with other pigments in raw linseed oil vehicle is not durable on outdoor exposure, this paint was selected as a paint that would not be durable, and could readily be described, so that it could be duplicated. The other two paints were prepared by using the same vehicle as in the lithopone paint, but selecting in one case a single pigment and in the other a mixed pigment that were reasonably sure to be more durable than the lithopone paint. The accelerated exposures have been completed. Eventually the bureau hopes to have results of outdoor exposures of the same paints. To date, however, there is practically no information on these outdoor exposures and at this time it would be impossible to recommend any particular formulas, were that the object of the test, since the most important tests—outdoor exposures—are still incomplete.

CALIBRATION OF IONOMETERS FOR X-RAY STANDARDIZATION.

THE solution of the problems of X-ray standardization has arrived at a point where the Bureau of Standards is in a position to calibrate ionometers for the public.

The discrepancies between different standardizing laboratories in regard to the magnitude of the *r*-unit are now small enough to be neglected in connection with medical applications. The basis for this conclusion is the agreement within 2 per cent., recently obtained by a comparison, through the coöperation of Doctor Glasser, of two instruments from

the Cleveland Clinic with those used by the Bureau of Standards; and the fair agreement found earlier between the Cleveland Clinic instruments and those of the Reichsanstalt of Germany. Further comparisons will, of course, be made between the different laboratories.

An ionometer sent to the bureau may be calibrated for any condition of voltage and filtration desired although all such instruments, to be accepted for calibration, must be adequately controlled by such means as radium or uranium oxide. This will minimize the possibility of undetected damage occurring to the ionometers in transit.

The following procedure is necessary to avoid delays and complication; notify the Bureau of Standards as far as possible in advance of the shipment of the instrument, giving at the same time the type of instrument, date of purchase, method of controlling its calibration, type of X-ray machine with which it is used, peak voltage applied to the X-ray tube as actually measured with a sphere-gap, and the half-value layer in copper or the effective wave-length of the radiation with an explanation of how the measurement was made.

If the ionometer has been previously calibrated a copy of the calibration should be supplied in order to complete the bureau's records for every instrument it handles.

The cost of this service will depend upon the number of calibrations made for each instrument. A schedule of fees may be had upon request after sending in the information required above in addition to stating the number of calibrations desired.

TENTATIVE STANDARD LABORATORY PROCEDURE FOR RUBBER TESTING.

In *The Rubber Age*, January 25, 1930, page 429, there appears the "Outline of Tentative Standard Laboratory Procedure for the Preparation and Physical Testing of Rubber Samples" as a report of the physical testing committee of the Rubber Division, American Chemical Society. The report presents a standard procedure for all details connected with the mixing of the raw compounds, their vulcanization, and the control of all factors such as time of mixing, humidity and temperature which might in any way affect the stress-strain relation of the final product.

This report embodies the results of research work carried on at the Bureau of Standards in coöperation with the physical testing committee since January, 1927, and it is expected will do much toward obtaining more uniformity in general laboratory procedure in the industry.

TESTS OF THE ARLINGTON MEMORIAL BRIDGE.

TESTS are being made by the Arlington Memorial Bridge Commission and the Bureau of Standards on one of the arch spans of the Arlington Memorial Bridge across the Potomac River at Washington, D. C.

The purpose of the tests is to obtain data on the structural behavior of the arch during and after its construction. Arch No. 7, the one selected for testing is the second arch to the west of the bascule draw span and is one of eight similar concrete barrel arches which with the draw span constitute the main portion of the bridge structure. Arch No. 7 was designed as a symmetrical, fixed end reinforced concrete arch having a clear span of 177 feet, 4 inches a rise of 27 feet, 9 inches, and a barrel width of 94 feet between the outside of granite voussoir block facings.

The instruments used in the tests include copper-constantan thermocouples and resistance thermometers for measuring the concrete temperature at various points. The thermocouples are connected to a multiple point potentiometer recorder. The rotations and deflections of the back of the arch barrel are measured by means of clinometers on a continuous line of clinometer stations extending over the extrados and a portion of each supporting pier. Readings are also made on several lines of clinometer stations situated transversely on the extrados. The strains in the extrados of the arch are measured by means of strain gages on ten groups of strain-gage stations. The concrete strains near the intrados are measured by means of electric telemeters, one of which is located radially beneath each group of strain-gage stations.

The temperature of the concrete in the thicker portions of the arch barrel reached a maximum of 140° F. within a period of 24 to 36 hours after being placed.

Under the assumption of no relative change in pier

elevation between the tests, it was found possible to measure the crown deflection with a probable error of two-thirds of one per cent. The clinometer accuracy was found to be sufficient to obtain significant deflection measurements corresponding to a uniform temperature change as low as two-thirds of one degree Fahrenheit.

The extrados bending strain was measured separately at selected points by both the clinometer and the strain gage and telemeter.

Glutathione. A reinvestigation of glutathione by FREDERICK GOWLAND HOPKINS (*J. Biol. Chem.*, 1929, *84*, 269–320) has demonstrated that this compound is a tripeptide, containing glycine, glutamic acid, and cysteine. It is unstable, thus, when its solution is boiled, glutamic acid and the diketopiperazine of cysteine and glycine are formed.

J. S. H.

Physical Constants of Guaiacol. T. S. CARSWELL (*J. Am. Pharmaceutical Asso.*, 1929, *18*, 995–997) has purified guaiacol by crystallization as its sodium salt, by precipitation as its magnesium salt, and by repeated fractional crystallization. The usual prismatic crystals were then found to have a crystallization point of 28.2° C., the needle-like form a crystallization point of − 3.2° C. The boiling point was 204.65° C. at a barometric pressure of 746.4 mm.

J. S. H.

The Potato as an Index of Iodine Distribution. ROE E. REMINGTON, F. BARTOW CULP, AND HARRY VON KOLNITZ (*J. Am. Chem. Soc.*, 1929, *51*, 2942–2947) have used the potato as an index to the distribution of iodine in the soil of South Carolina. Potatoes were grown at various localities in the state, and their iodine content was then determined, seventy-two samples were analyzed. The influence of the sea was apparent in potatoes grown in the coastal region. In all the other samples, the iodine content increased as the distance from the coast became greater. The chief source of the iodine in the soil is disintegrating granite rock of the Blue Ridge. Fertilizers may also serve as a source of iodine.

J. S. H.

Unusual Fermentation. In alcoholic fermentation, the fermented liquid rarely contains more than 16 per cent. of alcohol by volume. By special treatment during fermentation, the alcohol content may become about 18 per cent. JOHN R. EOFF, JR., HOWLAND BUTTLER, AND WILLIAM MELCHIOR (*Ind. and Eng. Chem.*, 1929, *21*, 1277–1279) have obtained as high as 19.4 per cent. of alcohol by fermentation of fruit juice, especially that of the pineapple, when the Tokay strain of yeast was used, fermentation was permitted to occur at a temperature of 15° C., and the must consisted, in part at least, of fresh fruit.

J. S. H.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, FEBRUARY 19, 1930.

THE regular monthly meeting of The Franklin Institute was called to order at eight-fifteen by Mr. Walton Forstall, Vice-President, who presided in the absence of the President.

The Chairman called upon the Secretary for a report of the business of the meeting. The latter stated that the minutes of the regular annual meeting held in January had been published in full in the February number of the Journal of the Institute, and moved that the minutes be approved as printed. The motion was seconded and adopted unanimously.

The Secretary reported that since the last report, the membership of the Institute has been changed in the following manner: By election—one Non-Resident Life Member; six Resident Members; four Non-Resident Members; two Student Members; by Death—two members.

The Secretary reported that at the meeting of the Board of Managers, held today, the following gentlemen were nominated by the Board for Honorary Membership in The Franklin Institute, and that these nominations are now submitted for the consideration of the meeting:

Sir William Bragg
Dr. John F. Stevens } Franklin Medalists for the year 1930

Dean M. E. Cooley, former Dean of the School of Engineering, University of Michigan, in recognition of his enlightened leadership in engineering education and his valuable services as a consulting engineer;

Dr. Henry Leffmann, scientist, teacher discriminating scientific critic, former Port physician of Philadelphia; stimulating as a professor, fruitful in research.

It was unanimously moved and seconded that these four gentlemen be elected to Honorary Membership in The Franklin Institute.

The Secretary then read the following lists of committee membership as those which had been appointed by the President of the Institute.

Committee on Library

C. W. Bates, Chairman
H. J. M. Creighton
F. L. Garrison
Clarence A. Hall
Joseph S. Hepburn
George A. Hoadley
Lionel F. Levy
Malcolm Lloyd, Jr.
M. M. Price
George F. Stradling

Committee on Meetings

Howard McClenahan, Chairman
James Barnes
George S. Crampton
T. G. Delbridge
W. H. Fulweiler
Arthur W. Goodspeed
Charles Penrose
George D. Rosengarten
W. C. Wagner
W. C. Wetherill

Committee on Museum

Hugo Bilgram, Chairman	John J. L. Houston
G. H. Benson, Jr.	Henry Leffmann
Arthur L. Church	Frederic Palmer, Jr.
Theobald F. Clark	H. H. Quimby
George A. Hoadley	W. O. Sawtelle

The Secretary announced further that the following gentlemen had been elected to membership in the Committee on Science and the Arts, by the Board of Managers, at its organization meeting in January.

G. S. Crampton	James D. Lee, Jr.	E. W. Smith
G. S. Cullen	K. G. Mackenzie	J. C. Trautwine, 3d
O. B. Evans	F. H. Rogers	J. A. Vogleson
J. Logan Fitts	J. M. Rusby	W. C. Wagner
F. V. Hetzel	S. H. Noyes	J. M. Weiss
G. A. Hoadley	W. O. Sawtelle	H. W. Welles
F. B. Evans, Jr.		W. C. Wetherill

He also announced that Mr. Henry B. Allen had been elected Chairman of the Committee on Science and the Arts for the present year.

There being no further business, the Chairman of the meeting, Mr. Forstall, presented the speaker of the evening, Dr. Albert W. Hull, Assistant Director of Research, General Electric Company, Schenectady, N. Y., who spoke on "Vacuum Tube Research."

Dr. Hull's speech was accompanied by brilliant experiments and was received with high enthusiasm and appreciation by a large audience. His talk was followed by vigorous discussion, after which the meeting adjourned with a rising vote of thanks to Dr. Hull for his delightful lecture, at nine-forty-five p.m.

HOWARD MCCLENAHAN,
Secretary

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, February 5, 1930.)

HALL OF THE COMMITTEE,
PHILADELPHIA, February 5, 1930.

DOCTOR THOMAS D. COPE *in the Chair.*

The Chairman announced the reëlection by the Board of Managers of seventeen members of the Committee to serve for three years.

The Special Committee on Nomination of a Chairman for the ensuing year reported as its choice Mr. Henry B. Allen. Mr. Allen was unanimously elected and took the Chair.

The proposed amendment of the Regulations to change the Stated Meeting of the Committee from the first to the second Wednesday of those months in which meetings are held was presented for final action and adopted.

The following reports were presented for final action:

No. 2900: Bailey Meters.

This report, recommending the award of the Edward Longstreth Medal to Mr. Ervin G. Bailey, of Fullerton, Pennsylvania, "In consideration of the practical inventive ability displayed in the development of his regulating and control devices and his measuring and recording instruments," was adopted.

No. 2917: Literature.

This report, recommending the award of the Louis Edward Levy Medal to Professor Floyd Karker Richtmyer, of Cornell University, for his paper on "Some Secondary Phenomena in X-ray Spectra" which was published in the September, 1929 issue of the JOURNAL OF THE FRANKLIN INSTITUTE, was adopted.

The following reports were presented for first reading:

No. 2909: The Work of Mr. Irving E. Moultrop in the Design and Construction of the 1200 lb. Pressure Edgar Station.

No. 2917: Work of Doctor Theodore Lyman of Harvard University.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, February 19, 1930.*)

NON-RESIDENT LIFE.

MR. WILLIAM GAERTNER, Manufacturer of Scientific Instruments, 1201 Wrightwood Avenue, Chicago, Illinois.

RESIDENT.

MR. FRANK B. EVANS, JR., Assistant Vice-President, Bell Telephone Company of Pennsylvania, 1835 Arch Street, Philadelphia, Pa.

MR. HARRY F. FLYNN, Civil Engineer, United States Engineers Office, 1109 Gimbel Building, Philadelphia, Pa. For mailing: 22 North Clifton Avenue, Sharon Hill, Pa.

MR. DAVID A. FOGELSANGER, Physical Research, Leeds and Northrup Company, Philadelphia, Pa. For mailing: 422 West Chelten Avenue, Philadelphia, Pa.

MR. W. D. GEISER, Electrical Tester, 2301 Market Street, Philadelphia, Pa. 4630 North Eleventh Street, Philadelphia, Pa.

MR. WILLIAM H. HOFFMAN, Industrial Salesman, Hajoca Corporation, 120 South Thirtieth Street, Philadelphia, Pa. For mailing: P.O. Box 303 Narberth, Pa.

MR. HARRY B. RUMRILL, Special Accountant, Pennsylvania Railroad Company, 15 North Thirty-second Street, Philadelphia, Pa. For mailing: Berwyn, Pa.

MR. SARKES TARZIAN, Atwater Kent Manufacturing Company, 4700 Wissahickon Avenue, Philadelphia, Pa.

DR. MAX TRUMPER, Physiological Chemist, 921 Medical Arts Building, Philadelphia, Pa.

NON-RESIDENT.

DR. JOSEPH C. BOYCE, Instructor in Physics, Princeton University, Princeton, N. J. For mailing: 30 Nassau Street, Princeton, N. J.

MR. JAMES A. DUNCAN, Physicist and Engineer, 55 Johnson Street, Brooklyn, N. Y.

DR. JOHN H. PERRY, Research Chemical Engineer, E. I. duPont de Nemours and Company, Experimental Station, Wilmington, Delaware.

MR. W. L. REINHARDT, Chief Engineer, Willard Storage Battery Company, 246–286 East 131 Street, Cleveland, Ohio.

STUDENT.

MR. J. ALEXANDER JERVIS, JR., Student, Drexel Institute. For mailing: 141 North Pennock Avenue, Upper Darby, Pa.

MR. W. BIDDLE WALKER, Student, 2103 Belevedere Avenue, Oakmont, Pa.

CHANGES OF ADDRESS.

MR. IRVING H. CROWNE, 1260 North Flores Street, Hollywood, California.

MR. FARNUM F. DORSEY, 205 Garfield Place, Maplewood, N. J.

MR. WILSON. N. DURHAM, President, Durham and Company, Inc., Radio Engineers, 2006 Chestnut Street, Philadelphia, Pa.

MR. S. B. ECKERT, Sun Oil Company, 1608 Walnut Street, Philadelphia, Pa.

MR. HERBERT A. ERF, 1500 Leader Building, Cleveland, Ohio.

MR. CHARLES F. FORSTALL, c/o The Reading Company, 2821 Richmond Street, Philadelphia, Pa.

MR. WALTER GRAF, 422 Roslyn Avenue, Glenside, Pa.

MR. DANIEL O. LANDIS, 7231 Walnut Street, Upper Darby, Pa.

MR. JOHN C. LYNCH, P. O. Box 93, Stockbridge, Mass.

MR. H. W. MAJOR, 155 Chambers Street, New York City.

MR. RICHARD C. MCCALL, "The Lenox," 1301 Spruce Street, Philadelphia, Pa.

MR. MATTHEW F. MAURY, c/o West and Company, 1511 Walnut Street, Philadelphia, Pa.

DR. E. F. NORTHRUP, Princeton, N. J.

MR. JOHN S. PFROMMER, 157 South Main Street, Sellersville, Pa.

MR. ROBERT RIDGWAY, 250 Hudson Street, New York City.

MR. ANTHONY W. ROBINSON, 780 College Avenue, Haverford, Pa.

MR. J. M. RUSBY, Spruce Hill Apartments, 4400 Spruce Street, Philadelphia, Pa.

MR. BENJAMIN SHACKELFORD, 377 Springdale Avenue, East Orange, N. J.

MR. A. T. SMITH, 12 West Fifty-sixth Street, New York City.

MR. I. MELVILLE STEIN, 3128 West Coulter Street, Germantown, Phila., Pa.

DR. J. HUNT WILSON, 532 Clinton Street, Easton, Pa.

MR. WILLIAM ZIMMERMAN, 524 Cresheim Valley Road, Chestnut Hill, Phila., Pa.

NECROLOGY.

Wilfred Lewis.* What I shall say this afternoon will deal more especially with the association of Wilfred Lewis with The Franklin Institute and its Committee on Science and the Arts to which he gave a valued and unstinted service.

* Remarks by Dr. George A. Hoadley at a Memorial Service held February 15, 1930 in the First Unitarian Church, Philadelphia.

Mr. Lewis became a member of the Institute in November, 1875 and remained a member during his life-time. He became a member of the Committee on Science and the Arts in 1897 and was so valuable a member that he was made its Chairman for the year 1912–13.

During that year many investigations of inventions and new devices were carried on by Sub-Committees appointed by him. He was well qualified to make these appointments because of his long acquaintance with the members; he became familiar with the special fields in which each one was interested and by such knowledge was able to assign to each subject for investigation a group of men who were well qualified to study the subject and form a correct estimate of its value to either science or one of its applications and also to determine what recognition should be recommended for the inventor.

As a result of these investigations an Elliott Cresson Gold Medal was awarded to Emile Berliner of Washington, D. C. for his improvements in "Telephony and Sound Reproduction" and a similar award was made to Alexander Graham Bell for his "Electrical Transmission of Speech."

When we consider the application of these fundamental inventions to the present marvelous use of the telephone in its many fields, we can begin to appreciate the foresight of Wilfred Lewis and his committees in their recognition of the practical value of these inventions and their usefulness to mankind.

A number of other Cresson Medals were awarded during the Chairmanship of Mr. Lewis for outstanding inventions or discoveries by men whose names have become well known for the additions they have made to our knowledge of science or of its technical applications.

Among these men were: Sir William Crookes, a noted English scientist, for his "Discoveries in Chemistry"; Doctor Emil Fisher, distinguished for his contributions to organic chemistry; Professor Albert A. Michelson, the well known physicist who has recently made the most accurate determination of the velocity of light, for his "Work in Physical Optics"; Mr. Alfred Noble, a nationally known engineer for his "Achievements in Civil Engineering"; Lord Rayleigh for his "Researches in Physical Science"; Charles P. Steinmetz for his outstanding "Work in Electrical Engineering"; Elihu Thomson, former science teacher in the Central High School of this city for his "Industrial Applications of Electricity"; and Sir J. J. Thomson for his "Leading Work in Physical Science."

This list of awards could be lengthened and not exhaust the names of workers in their chosen fields, fields in which Mr. Lewis was interested and in which he did much to secure for these workers the recognition they deserved.

In addition to acting on the Committee that recommended awards to others, Mr. Lewis was himself the recipient in other years of two Edward Longstreth Medals, one for his invention of an "Inertia Indicator" in 1899 and the other for his "Shockless Jarring Machine" in 1926.

During the fifty-four years of Mr Lewis' membership in The Franklin Institute and particularly during those years in which he was an active member of the Committee on Science and the Arts, his fellow members learned to know him intimately and to know that his judgment in regard to mechanical devices and inventions could be relied on. He was loyal to the purposes and aims of the Institute and showed that loyalty by the way in which he gave of his time and experience in working for its interests.

He was an inspiration to the younger men who joined the Committee on Science and the Arts, giving freely of his wide experience and knowledge, encouraging them to give the best that was in them, advising a thorough study of the various devices submitted for their examination and teaching them to separate from the rest, those that were worthy of recognition.

The value of such a life to an institution is beyond estimate and its influence endures far beyond the limit of a man's life.

It is through the unselfish work of such men as Wilfred Lewis that The Franklin Institute has reached its position of high standing among scientific men both at home and abroad, and to such men we extend our fullest appreciation of what they have accomplished.

Dr. George F. Baker, Philadelphia, Pa.
Mr. Charles T. Mason, Sumter, S. C.
Mr. Luther M. R. Willis, Baltimore, Md.

LIBRARY NOTES.

RECENT ADDITIONS.

AHRONS, E. L. The British Steam Railway Locomotive 1825–1925. 1927.

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ANDERSON, J. E., AND HERMAN BERNARD. Audio Power Amplifiers. 1929.

AUDEN, H. A. Sulphuric Acid and its Manufacture. 1930.

Beilsteins Handbuch der organischen Chemie. Vierte Auflage. Erstes Ergänzungswerk. Dritter und vierter Band. 1929.

Bulletin Almanac and Year Book. 1930.

Chemiker-Kalender 1930. Two volumes.

CONDON, EDWARD U., AND PHILIP M. MORSE. Quantum Mechanics. 1929.

CURRIER, CLINTON HARVEY, AND EMERY ERNEST WATSON. A Course in General Mathematics. 1929.

DEHAAS, WALTER (pseud. HANNS GÜNTHER). Fünfsprachenwörterbuch für Radioamateure. 1924.

EATON, HERBERT N., AND OTHERS. Aircraft Instruments. 1926.

EMSLEY, H. H., AND WM. SWAINE. Ophthalmic Lenses. 1928.

EVE, A. S., AND D. A. KEYS. Applied Geophysics in the Search for Minerals. 1929.

FILGATE, JOHN T. Theory of Radio Communication. 1929.

HARDING, LOUIS ALLEN, AND ARTHUR CUTTS WILLARD. Mechanical Equipment of Buildings. Volume I. Heating and Ventilation. Second edition, revised and enlarged. 1929.

HELLBUSCH, ERWIN. Deutsch-englisch-französisch-spanisches Fachwörterbuch für den Chemikalienhandel und die anschliessenden Gebiete. 1921.

HENNEY, KEITH. Principles of Radio. 1929.

HERZOG, H. O. Wörterbuch der modernen Maschinenwerkstatt. Two volumes. 1926.

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KUHLMANN, JOHN H. Design of Electrical Apparatus. 1930.

LAWRIE, L. G. Textile Microscopy. 1928.

London, Metropolitan Water Board. Twenty-Sixth Annual Report. Year ended 31st March, 1929.

MARSHALL, C. F. DENDY. Centenary History of the Liverpool and Manchester Railway. 1930.

MAYER, A. W. Chemisches Fachwörterbuch: Deutsch-Englisch-Französizch, für Wissenschaft, Technik, Industrie und Handel. Erster Band. 1929.

MIDGLEY, EBER. The Finishing of Woven Fabrics: Technical Factors and Principles. 1929.

National Association of Sheet Metal Contractors—Trade Development Committee. Standard Practice in Sheet Metal Work. 1929.

OWENS, JAMES W. Fundamentals of Welding: Gas, Arc and Thermit. First edition. 1923.

PAESSLER, JOHANNES, UND R. LAUFFMANN. Wörterbuch der wichtigsten in der Lederindustrie angewandten Fachausdrücke. Deutsch-Englisch und Englisch-Deutsch; Deutsch-Französisch und Französisch-Deutsch. Two volumes. 1912–1913.

PAGÉ, VICTOR W. The Modern Gasoline Automobile. 1928 edition, thoroughly revised.

Photograms of the Year 1929.

ROEMER, THEODOR. Handbuch des Zuckerrübenbaues. 1927.

ROHR, MORITZ VON. Die binokularen Instrumente. Zweite Auflage. 1920.

SANDERSON, CLARENCE HERBERT. Electric System Handbook. First edition. 1930.

SATTELBERG, O. Englisch-Deutsches und Deutsch-Englisches Wörterbuch der elektrischen Nachrichtentechnik. Two volumes. 1925–1926.

SCHULZ, HANS, AND KARL RADICKE. Fremdwörterbuch für die Optik. 1929.

SEEM, WARREN P. Raw Silk and Throwing. First edition. 1929.

THEWS, EDMUND RICHARD. Metallurgy of White Metal Scrap and Residues. 1930.

THUROW, WILLY H. Englisch-Deutsches und Deutsch-Englisches Wörterbuch der Chemie. Teil I. Englisch-Deutsch. 1929.

TURNBULL, H. W. The Theory of Determinants, Matrices, and Invariants. 1928.

UEHLING, E. A. Heat Loss Analysis: the Key to Economic Boiler Operation. First edition. 1929.

United States Department of Commerce. Commerce Yearbook. Two volumes. 1929.

VEAL, T. H. P. The Disposal of Sewage. 1928.

VILLAT, HENRI. Leçons sur l'Hydrodynamique. 1929.

West Virginia Geological Survey. County Reports. Pocahontas County. Two volumes. 1929.

WIEN, W., AND F. HARMS, editors. Handbuch der Experimentalphysik. Band 5, and Band 24, 2 Teil. Two volumes. 1930.

WIESNER, JULIUS VON. Die Rohstoffe des Pflanzenreichs. Vierte Auflage. Bd. 1–2. 1927–1928.

WOODHOUSE, THOMAS. The Preparation and Weaving of Artificial Silk or Rayon. 1929.

World Almanac and Book of Facts. 1930.

WYNN, A. E. Design and Construction of Formwork for Concrete Structures. No date.

BOOK REVIEWS.

INTRODUCTION TO PHYSICAL OPTICS. By John Kellock Robertson, F.R.S.C., Professor of Physics, Queen's University, Kingston, Canada. vii–422 pages, illustrations, 8vo, cloth. New York, D. Van Nostrand Company, Inc., 1929. Price, $4.00.

There is available a considerable collection of comprehensive modern texts on physical optics, and the author of this volume regards the field represented by those works and by books of a strictly elementary character as already well covered. In an intermediate stage of study which he describes as neither elementary nor advanced, he finds another text is needed. The work is specifically addressed, on the one hand, to those students who at the outset of an intensive study of physics are laying a firm foundation for subsequent work on the theory of optics, and on the other hand, to those who will major in some other branch of physics for which a sound general knowledge of modern views of light will be essential.

A generally characteristic feature is the obvious aim to insure a thorough assimilation of the topics discussed, and to that end the purely analytic matter is reinforced by numerous quantitative discussions and explanatory references which arouse interest and stimulate a capacity for sustained effort. Despite the profound changes which twentieth century progress has wrought in our ideas of matter and the propagation of radiant energy, the century-old universally accepted wave theory still dominates. If it is to pass or at least be extensively modified by the discontinuous emission of radiant-energy which is postulated by the quantum theory, it has proved a working hypothesis of incomparable value about which the magnificent structure of modern physics has been built. The new facts of physics, far reaching as they are, have not yet modified the practice of the science as a whole. Thus in this up-to-date work we find the subject matter developed according to the customary wave-theory and a continuous wave-front with appropriate digressions upon those phenomena accountable only on the new atomistic theory and on the discontinuous character of the interchange of radiant energy.

The development of physical theory, particularly in its modern forms, often carry the investigator into the most tortuous paths of mathematical science.

The author bears in mind that his work is not for a trained mathematician but for the student whose mathematical attainments are limited. He accordingly limits the deductive features to those which can be approached by algebraic and trigonometric processes; results which require transcendental processes are stated without formal demonstration. Such a program for a first reading of a subject is obviously consistent with the purpose of acquainting the reader with the great quantity of qualitative information which is associated with the subject. The author, however, is insistent on the acquisition of a facility in dealing also with quantitative results. Particularly is this stressed by the inclusion of an unusually complete introductory study of wave-motion.

Reflectors, lenses, prisms and the telescope, including thick lenses, receive the first attention and are very adequately treated by the uniform application of the Huygens principle, thus maintaining in the derivations the physical concepts of light propagation. This occupies about one-third the volume. The remainder covers more strictly the realm of physical optics, dispersion spectra, interference, polarization and lastly the electro-magnetic theory of light and the modern development of the "new physics." This division of the subject contains many half-tone photographs of spectra, interference, and diffraction phenomena. These are a distinct help to the student who may, but probably will not, have the opportunity of viewing them all "in the flesh." The modern theories and developments are grouped in the final chapters and constitute an illuminating outline of those important advances. Since they have not yet modified the old and still useful working hypothesis of the school-book, they are rather matters for the post-graduate student and research specialist. The book is indeed a thorough introduction which closely approaches "reduction to practice."

L. E. P.

INTRODUCTION A UNE CONNAISSANCE SCIENTIFIQUE DES FAITS MUSICAUX. Par Pius Servien. 54 pages, 8vo. Collection du Suggestions Scientifiques publiee sos la direction de Leon Brillouin. Paris, Librarie Scientifique Albert Blanchard, 1929. Price 7.50 francs.

This pamphlet is one of a collection of scientific suggestions published under the direction of Léon Brillouin. In this particular instance the author seeks to apply the approved methods of scientific analysis and investigation to the field of music. His introductory statement is to the effect that music quite often has a soothing and quieting effect; that this is due to the presence of a symmetrical relation between tone intervals and duration of the notes and that such a symmetry may be demonstrated graphically by plotting time as the abscissa against the intervals (in semitones) as the ordinate. Music is classified either as cyclic or linear. Cyclic music tends to calm the agitated mind; graphically it shows an enclosed space and is best expressed by the diatonic scale. On the other hand, linear music is rather similar to a continuous running chain, there is no letdown in the tension, there being a constant progression from one transformation to the next. Such music is best fitted to the chromatic or atomic scale.

Part Two deals with the recognition of themes. Here is outlined the several types of transformation a theme may undergo and a graphical analysis made of the themes in "Tristan and Isolde." Following this the author discusses the relations of timbre, number of notes, length of intervals, time and intensity and the part played by each in the various transformations of a given theme.

The third and final section is occupied with a mathematical determination of the number of possible series of notes which would furnish transformations in both the diatonic and chromatic scales.

The author discloses the fact that he possesses an intimate knowledge of the principles of music and has applied the scientific methods of attack in a very able manner.

T. K. CLEVELAND.

SOAP FILMS. A study of molecular individuality. By A. S. C. Lawrence with a foreword by Sir William Bragg, F.R.S. xi–141 pages, illustrations, 8vo, cloth. London, G. Bell and Sons, Ltd., 1929. Price 12s. 6d.

The brilliant researches of the late Sir James Dewar constitute one of the modern classics in physical science. Among his many activities, as recorded in his collected papers, will be found a lengthy investigation of the physics of soap films which was conducted in his usual masterly fashion. The author of the present volume for many years acted as Professor Dewar's assistant, and naturally enjoys most favorable facilities to compile a monograph on soap films from the knowledge acquired of the technique of the subject in that famous laboratory as well as from his own views.

The subject is developed from the fundamental problem of why do soap films exist at all, what is a soap film and how does it manage to exist? In answering these questions we are first given an explanation of the nature of surface tension and its relation to the soap film structure, one of whose dimensions is of the same order as those of its constituent molecules, where the reality and behavior of individual molecules is stressed. The chemistry of soap is treated at considerable length and the behavior of its molecules when dissolved in water. Continuing to soap films, a full account of their properties follows, especially of those discoveries made in the last few years. The variety of the metamorphoses of the soap film, which the author states are described fully for the first time show, he points out, how many phenomena must be included in any adequate theory of its nature and behavior. Appendices on the preparation of oleic acid, and on the color and thickness of soap films are added. The work is thoroughly documented and fully illustrated with diagrams, and halftones of films, some of them in color, and of apparatus. The book is distinctive in treating the subject in harmony with the methods of recent developments in physical science.

L.E. P.

OPTISCHE METHODEN DER CHEMIE. Von Fritz Weigert, a. o. Professor and der Universität Leipzig. 632 pages, illustrations, plates, 8vo. Leipzig, Akademische Verlagsgesellschaft m. b. H., 1927. Price, bound, 38 marks.

There is no doubt but that every day and in every way the chemist tends more and more to rely upon optical methods to aid him in his researches, investigations, analyses and control of manufacturing processes. The researcher would feel lost indeed were he denied use of the spectroscope, spectrophotometer, microscope, ultramicroscope, photometer, refractometer, polarimeter, colorimeter, nephelometer and a great variety of other apparatus involving the application of optical methods. The colloid chemist must have his ultramicroscope, the textile chemist and dyer their spectrophotometer, the control chemist his refractometer, optical pyrometer, photometer, polarimeter, the analyst his colorimeter, nephelometer and interferometer.

Professor Weigert in the preparation of his book, "Optische Methoden der Chemie," has assumed the task of informing the chemist concerning the extensive application of optical methods to his particular branch of science; of describing the various types of apparatus and operation of the same; and of furnishing a critical analysis on the accuracy and interpretation of the results obtained. Although the author points out that his material has been drawn to a great extent from his personal experience and knowledge, he appears to have covered the whole field quite comprehensively, laying particular stress upon the accuracy, limits of the measurements and their critical application in the desired fields of investigation.

The first three chapters involve a general discussion of optical instruments and their accessories. Various types of illumination are described and their characteristic radiations enumerated. Chapter Three lists a great variety of color filters and the absorption spectrum peculiar to each. Chapters five and six contain descriptions of various types of spectrometers and photometers, particular attention being paid to the application of photoelectric cells to photometry work. Chapter seven deals with spectrophotometry, a subject which always has received considerable attention among theoretical chemists in particular. For, by this method are determined absorption spectra extinction coefficients and color composition of many substances. Chapter eight on colorimetry and nephelometry deals with an application of optical methods not yet come into its own. That their adoption is becoming more general is evidenced by the recent works on the subject of nephelometry. The chapter on color measurement draws a comparison between colors as interpreted by the eye and as analyzed by the spectrophotometer. The subject of energy measurement is first discussed theoretically and is followed by a description of methods which are based upon the use of the thermopile, bolometer, radiometer, photometer, etc. Chapter nine discusses the subject of photochemical measurements at considerable length. It includes an explanation of Einstein's Law of Photochemical Equivalence, and exhaustive directions for conducting investigations of photochemical reactions. The subjects Microscopy and Ultramicroscopy, Refraction and Analysis of Polarized Light are treated with the usual thoroughness. Ultramicroscopy will be of particular interest to colloid chemists, while refractometric determinations are of almost universal application. A separate chapter is accorded the subject of luminescence as evidenced by fluorescent and phosphorescent phenomena. The book is supplied with an appendix which contains tables listing the spectra of several elements, the iron arc and element lines used in the analysis of ultraviolet spectra. There is also a table of the Newtonian colors for air films and silver films. Following the author and subject indexes will be found sixteen pages containing forty-two figures with accompanying explanations.

T. K. CLEVELAND.

GERMAN FOR CHEMISTS. By Louis De Vries, Ph.D., Iowa State College. 180 pages, illustrations, 8vo. Easton, The Chemical Publishing Company, 1929. Price, $3.00.

Several books of this type are now available to English-speaking students. Such works are valuable because they combine instruction in the German language

with information in science, especially articles by leading chemists from which the student may get inspiration as to methods of research in vogue to-day, and also by some contributions of older date get an insight into the development of modern chemistry.

Any detailed criticism of the book is impossible. All that can be said would be in reference to the selection of articles, to which the reviewer makes no objection. Much variety is shown ranging over the entire field of pure and applied chemistry. A touch of the romantic is given in the reprint of Kekulé's essay on the "Benzoltheorie" published in 1890, presumably on the occasion of the celebration of the twenty-fifth anniversary of the publication of his theory of the ring-structure of the carbon nucleus of benzene.

To insure that students in chemistry and physics shall have a good reading knowledge of French and German is no easy task as American Colleges are now operated. There is no question that many English-speaking persons find German somewhat uncouth and are by certain social relations not friendly to its study, while French though enjoying considerable popularity is irregular in pronunciation and many of its sounds are so different from those of English as to require much practice for proper use. As a rule the teachers of German and French in our colleges are not interested in science, and books of the type now under review are not used by them. Anyone who has observed the imperfect acquaintance that most students get of these two important languages must regret that much more strict requirements are not enforced. To-day the two culminating degrees are Ph.D. and Sc.D. No student should receive either of these who cannot read both French and German at sight.

It is also somewhat unfortunate that most of those students who acquire a good reading knowledge of either language stop at the scientific literature, losing the culture and pleasure of the general literature, especially in German with its rich supply of plays for "tragedy, comedy, pastoral, pastoral-comical," etc., as well as fiction, poetry, philosophy and humor.

HENRY LEFFMANN.

DIATOMACEOUS EARTH. By Robert Calvert. American Chemical Society Monograph Series. 251 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1930. Price, $5.00.

The American Chemical Society continues to keep chemists under obligation for valuable contributions to the literature of applied chemistry and the Chemical Catalog Company supplements the service by presenting the contributions in excellent form. In the present volume we have another of the numerous treatises issued under these auspices. The subject "Diatomaceous Earth" seems at first sight hardly sufficient in scope or importance to require a sizable volume, but a perusal of the book shows a remarkable range of usefulness in these peculiar deposits. The present reviewer remembers very well the days when about the only scientific interest in this material was by amateur microscopists who used it for selecting and mounting the many species to display the forms, always striking and often very beautiful. The markings upon them were at one time used for testing the resolving power of objectives. The deposits were often erroneously called "infusorial earth." The author of the book calls attention to this and other erroneous names given to the material.

It is certainly impressive to note the many uses to which diatomaceous earth is now applied and to note the enormous extent of available deposits and the amount of material mined. The application that will most astonish chemists is as a catalyst. A specific instance is in the hydrogenation of oils, but it must be confessed that the diatoms here are rather a support for the catalyst than the active agent. The descriptions of the application show that the nickel, which is the real catalyst, is deposited on the surface of the diatoms, their minuteness serving to promote the subdivision of the metal. A further service is the fact that the diatomaceous earth can be used as a filtering material to remove the nickel from the fat after the process of hydrogenation is accomplished.

Diatomaceous earth has been found to be serviceable as a source of silicates. This would hardly be supposed *a priori*, for, though minute, the forms are definite and firmly composed. It appears, however, that solutions of the fixed alkalies or even milk of lime will produce silicates, the reaction being especially vigorous when the water suspension is warmed. By fusion, silicates are, of course, readily formed, even such salts as borates being decomposed at high temperatures. Portions of the earth mixed with rosin, lime and basic slag are used to make the "clay pigeons" that are much used to-day in trap-shooting in substitute for the cruel practice of "live shoots" of earlier years.

The book is abundantly illustrated by photogravures of apparatus and plant installations, as well as diagrams and tables. It is well printed on good paper. The preparation of the text reflects much credit on the author and the printing on the publishing company. It is another valuable American contribution to the literature of applied chemistry.

HENRY LEFFMANN.

DIELECTRIC PHENOMENA IN HIGH-VOLTAGE ENGINEERING. By F. W. Peek, Jr. Third edition. xv–410 pages, illustrations, 8vo, cloth. New York, McGraw-Hill Book Company, Inc., 1929. Price, $5.00.

Much progress has been made in recent years in the study of the magnetic and dielectric fields of charged conductors and a large amount of quantitative information has accumulated on the properties of dielectrics under transient as well as under stable conditions in a form applicable to the problems of design of electrical apparatus and transmission lines. In these days of high-voltage and high-frequency phenomena, and the necessity of protecting transmission lines from the incidental and highly destructive effects of atmospheric electrical disturbances, the present work, the first edition of which appeared in 1915, has proven a valuable and timely addition to the literature of electrical engineering.

In this third edition, the author has made a thorough revision of the work, and has included new field and laboratory data which have become available. The new material comprises studies of corona, investigations of the impulse breakdown of gases, of solid and liquid insulation, and spark-over voltages of gaps and insulators at very high voltage. In view of the growing importance of lightning, that subject is treated in a separate chapter. About fifty pages are devoted to energy relationships of electric and magnetic fields, a mathematical study of the dielectric field and dielectric circuit, and the electron theory. From that point the work deals with the application of those principles to the analysis

of observed phenomena and its translation into applicable design data. Visual corona, its occurrence under a variety of conditions and its characteristics are examined at length. Under the caption "spark-over" the initial complete breakdown of a gaseous dielectric is given an equally lengthy analysis. In logical order come the consideration of the losses due to corona, corona and spark-over in oil and liquid insulators, and an examination of numerous solid insulations. In a thorough chapter on lightning the recent work of some of the most notable contributors in that field has been utilized. In general the work is well documented by numerous references to original sources. A final chapter, a reduction to practice, deals with the design of apparatus where solid liquid and gaseous insulations enter in combination. An extensive collection of tabulated data is given in an appendix. Illustrations are of a high order of excellence and numerous. The work is indeed a fine piece of book-making which reflects credit alike to author and publisher.

L. E. P.

METALLURGY OF WHITE METAL SCRAP AND RESIDUES. By Edmund Richard Thews, consulting metallurgist. 383 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1930. Price, $5.50.

"Scrap" is a characteristic word of our present day. Everywhere we find scrap; scrap paper, scrap wood, scrap metal, scrap automobiles and in some of our largest cities a very considerable collection of scrap houses. A good deal of attention has been paid to devising methods for utilizing some of these forms of scrap. Paper scrap, for example, is a very abundant and troublesome item in the management of our city streets. Unfortunately the great bulk of paper material is of such inferior character that it does not pay to recover it even under the cheapest methods.

The book in hand is devoted to the chemistry and metallurgy of a very important class of scrap, that from some of the metals which are extensively used in our household life and which are being abandoned in consequence of injury during use. Tin, lead and zinc are among the most important, but aluminum is rapidly becoming familiar in scrap form. Notwithstanding the attention that has been given to this subject it appears that no comprehensive description of the methods for treating these materials has been furnished to American literature, and the present volume is offered to fill this want. A small handbook on the subject was published in England about ten years ago and the author contributed an essay in German lately.

In the preparation of the volume the author has given, not only the important facts directly related to the treatment of white metal wastes, but also the elementary principles of such procedures. Aluminum has been included in the work and attention has been paid to the requirements of small and medium size plants, conducted independently as well as to the operations of large works which carry on for themselves methods for recovery. Many points accessory to the operations have been included. As the first publication in book form in United States upon the subject, it will be a welcome addition to the library of the works-chemist and the metallurgist. The author invites suggestions from users of the book and it is to be hoped that he will receive such aid, for while it is a comprehensive volume, representing a large amount of experience and labor, there may be

further data in possession of those engaged in such operations. The work is liberally illustrated, printed clearly on good paper, and represents another gratifying evidence of the advance in applied chemistry in this country.

H. L.

MODERN METHODS OF COCOA AND CHOCOLATE MANUFACTURE. By H. W. Bywaters, D.Sc., Ph.D., A.R.C.Sc. and F.I.C. 316 pages, illustrations, 8vo. Philadelphia, P. Blakiston's Son & Company, Inc., 1930. Price, $6.00.

South America has given to the world a considerable number of products that make for human safety and comfort. From that half-continent of which the general aspect could be described as Bishop Heber describes Ceylon, we get chocolate, Peruvian bark, potato (sweet and white), tomato and coca leaves. Many other things might be mentioned. Of the four vegetable products, cacao, coffee, tea and maté, which contribute so largely to human enjoyment in the "cups that cheer but not inebriate," cacao is the only one that finds extensive employment in other fields than as a beverage. In the form of chocolate and also under the erroneous but usual name of cocoa, it administers to the "sweet tooth" of the nations. In the book before us we have a comprehensive account of the various forms of cacao as cultivated and gathered in different parts of the tropical world. The amount of material now produced is enormous. The importation into the United States alone amounts annually to several hundred thousand tons. With such a demand it follows that cultivation has been carried on on a large scale in many places where climate permits the satisfactory growth of the plant. The usual interference by various diseases, fungous and by the action the insects have, of course, developed remarkably and have been antagonized more or less successfully by research. A brief historical notice is given from which we learn that it was not until 1519 that the cacao tree was seen by Europeans on Cortez' expedition to Mexico. The word cacao is a modification of the native name adopted by the Spaniards. A preliminary section of about sixty pages gives an account of the growth of the bean, its structure, the varieties from different places, also methods of storage and the means of determining quality. In any great industry of this type other industries will be influenced for better or worse. The author points out that an enormous production of chocolate stimulates the sugar industry. The second section of nearly one hundred pages covers all the details of the manufacture of cocoa, by which term it is now generally agreed among manufacturers to indicate the powder, chocolate being the refined mixture of this with sugar with or without the cacao butter. It is important, by the way, to avoid the confusion between the chocolate-nut which is one name for the product here under consideration and the coconut which has often been confused with the chocolate nut on account of the incorrect spelling cocoanut. The products are entirely different. The chocolate-nut or cacao is derived from *Theobroma cacao*, the coconut from *Cocos nucifera*. Linnaeus probably found chocolate exceedingly pleasant and he dubbed the plant yielding it as Theobroma "food for the Gods." The coconut, the fruit of a palm, has no botanical or chemical analogy to the chocolate-nut. Cacao contains a well-marked nitrogenous principle closely analogous to caffein which occurs in coffee, tea and maté. It does not appear that we as yet know very much about the effects of these substances upon the human system but there is a good deal of

discussion upon the subject largely based upon the economic problems involved. The present book, as might be expected, shows some enthusiasm for the various forms of cacao as nutritious articles of food, but it seems to the reviewer that they are more of the nature of condiments than nutritives.

The book is liberally illustrated and is a comprehensive and useful contribution to a subject of great interest and importance. It is of British origin but seems to cover the general world-practice. HENRY LEFFMANN.

PUBLICATIONS RECEIVED.

Modern Methods of Cocoa and Chocolate Manufacture, by H. W. Bywaters. 316 pages, illustrations, 8vo. Philadelphia, P. Blakiston's Son & Co., Inc., 1930. Price $6.00 net.

German for Chemists, by Louis de Vries, Ph.D. 180 pages, 8vo. Easton, Penna., The Chemical Publishing Co., 1929. Price $3.00.

Handbook of Chemistry and Physics. A ready-reference pocket book of chemical and physical data. Fourteenth edition compiled from the most recent authoritative sources by Charles D. Hodgman, M.S., and Norbert A. Lange, Ph.D. 1386 pages, 12mo. Cleveland, Ohio, Chemical Rubber Publishing Company. Price $5.00.

The Balancing of Oil Engines, by W. Ker Wilson, B.Sc. 279 pages, illustrations, plates, 8vo. London, Charles Griffin and Company, Ltd., Philadelphia, J. B. Lippincott Company, 1929. Price $10.00.

Leçons sur la Théorie des Tourbillons, par Henri Villat. 300 pages, 8vo. Paris, Gauthier-Villars et Cie., 1930. Price 65 francs.

The Terminology of Physical Science, by Duane Roller. 115 pages, 8vo. Norman, University of Oklahoma Press, 1929. Price $1.00.

A Course in English for Engineers. Volume I, The Engineer's speaking and writing—in general, by Carl A. Naether, M.A., in collaboration with George Francis Richardson, Ph.D., with an introduction by J. A. L. Waddell, D.E., LL.D. 330 pages, 8vo. Boston, Ginn and Company, 1930. Price $2.60.

Vocational Mathematics, by Edgar M. Starr and Edwin G. Olds. 184 pages, illustrations, 12mo. Philadelphia, P. Blakiston's Son and Company, Inc., 1930. Price $1.25.

National Advisory Committee for Aeronautics. Technical notes: No. 330, Wind tunnel pressure distribution tests on a series of biplane wing models, part III, Effects of changes in various combinations of stagger, gap, sweepback and decalage by Montgomery Knight and Richard W. Noyes. 6 pages, illustrations, quarto. Washington, Committee, 1929. No. 331, Rate of heat transfer from finned metal surfaces. Progress report on investigations at aeronautical engineering department, Massachusetts Institute of Technology, by C. Fayette Taylor and A. Rehbock. 21 pages, illustrations, quarto. Washington, Committee, 1930.

For Home Lovers. A collection of original designs for dwellings containing from five to eight rooms. Publication prepared under the direction of Richard G. Kimbell, architectural advisor. 32 pages, illustrations, quarto. Washington, National Lumber Manufacturers Association, 1929.

CURRENT TOPICS.

The Ozone in the Atmosphere. D. N. HARRISON. (*Nature*, July 13, 1929.) It was only in 1920 that Fabry and Buisson at Marseilles made the first accurate measurements of the ozone content of the atmosphere yet their method has been so developed that ozone observations are made at least once a day at about six observatories in different parts of the earth.

The absorption spectrum of ozone has bands in the infra-red, a weak band in the visible spectrum and a very strong band in the ultra-violet that cuts off the solar spectrum entirely at 2900 A.U. The absorption band in the ultra-violet is used to determine the amount of ozone in the atmosphere, the amount of energy corresponding to this band that reaches the earth depending upon the quantity of ozone it has traversed before attaining the recording apparatus. A photographic method furnishes the intensity. The quantity of ozone in the air at any time is expressed as so many millimeters of the pure gas at normal temperature and pressure, for Europe generally about 3. This amount is, of course, scattered through a considerable depth of air. The method is of such accuracy that results can be given with three significant figures. The common unit used is .001 cm. so that 321 means .321 cm. of ozone. By several workers the height of the ozone layer is placed at 40–50 km. Its elevation seems to become greater with increasing quantities of ozone present.

The following discoveries were made at Oxford in 1925, (*a*) from day to day and even during the same day the quantity of ozone may vary greatly, (*b*) as the barometer goes up the quantity of ozone goes down and vice versa, and (*c*) a distinct annual variation exists with a maximum of about 330 in April and a minimum of 220 in October. "It is now certain that the various sequences of weather are accompanied by definite and characteristic changes in the quantity of ozone in the atmosphere above any given place." Confirmation of this important conclusion comes from measurements made at Oxford, Lerwick (Shetland), Abisko (Norway), Valentia (Ireland), Arosa (in the Alps), and Marseilles. There is a close connection between the ozone content and the air-currents that bring changes of the weather. In this the stratosphere, the isothermal region of the air beginning about 10 km. above the earth's surface, has the greatest importance. Air from the tropics is connected with low ozone values and polar air with high values. When a cyclone comes, the ozone content falls in front of the depression and rises after the center has passed. In an anticyclone the minimum value for ozone comes after the maximum of barometric pressure.

Up to the present time the highest and lowest values for ozone content recorded are 420 and 172. "The correlation coefficients between the amount of ozone and upper air conditions are among the highest found in meteorology. The correlation with surface pressure is high, but those with the pressure at heights of 9.14 km. are much higher and very high values are found for temperature in the troposphere and for the height of the base of the stratosphere; these are all negative while that for the temperature at 14 km. (in the stratosphere) is positive."

Since ozone is a minimum in the tropics and a maximum in polar regions it seems that sunlight does not change oxygen into ozone, or at best is not solely responsible for the change. The origin of ozone may lie in the aurora and high ozone values occur at times of magnetic disturbance.

G. F. S.

Some Problems of Cosmical Physics, Solved and Unsolved. LORD RAYLEIGH. (*Science*, July 26, 1929.) This is the address of the president of Section A, Mathematical and Physical Sciences, of the British Association for the Advancement of Science, at its meeting in Cape Town, July 24, 1929. A review is given of some subjects belonging both to astronomy and to physics.

After the first success in identifying the spectral lines of the sun and stars with those produced by terrestrial elements it was seen that there remained certain lines that could be produced in no known way by materials on the earth. The spectral autograph of helium was recognized in the sun about a generation before the gas itself was discovered on the earth. To go further from our planet there are lines in the spectra of nebulæ that have never been discovered on the earth, and they are not faint members of complex spectra but "stand out, bold and challenging, on a dark background, presenting a puzzle that was the more intriguing from its apparent simplicity." Through the work of Hartley, of I. S. Bowen in the United States and of A. Fowler it is now certain that one pair of the nebular lines come from doubly ionized oxygen and that other nebular lines have their origin in singly ionized oxygen and nitrogen.

For a long time a main problem was the explanation of the green line in the aurora, first seen by Ångström in 1868. Nothing else can be seen in any but bright auroras except this line. "For some years I took every available opportunity of looking at this spectrum, and never did so without a deep sense of mystery. The origin of the line was not in this case in the depths of space, but in our own atmosphere at the distance of a short railway journey from the observer. Yet an apparently exhaustive study of the spectra to be obtained

from terrestrial gases by the combined efforts of very many experimenters gave no clue to its origin." At last McLennan in Toronto got the line by sending heavy electric charges through mixtures of oxygen and helium or oxygen and argon. The green auroral line is without doubt due to the arc spectrum of oxygen but by no means all is clear about its production. In the sky it appears alone or with negative nitrogen lines, whereas in the laboratory it is accompanied by other oxygen lines and by lines of argon or helium. Red auroras are rare but still they do occur and the red line that dominates their spectra awaits explanation. The strong lines in the spectrum of the sun's corona present another unsolved problem.

Polar auroras are connected with magnetic disturbances and these have to do with the sun. The theory of Birkeland, extended by Störmer, is held to emit "localized streams of electrically charged particles from limited areas of its surface." This theory presents advantages in accounting for the occurrence of the aurora but artificial hypotheses have been attached to it. Recently it furnished the only conceivable explanation of a newly found fact. Hals in Holland sent out wireless signals 131 meters in length and after 13 secs. received echoes of them. Keeping in mind that the speed of such signals is 300,000 km. per sec. one sees that the surface reflecting the signals must be farther from the earth than the moon is. The reflector is identified by Störmer with the stream of corpuscles from the sun bent around by the earth's magnetic field. This is a daring conjecture indeed. G. F. S.

Stable Solutions of Sodium Thiosulphate. H. W. JONES (*Chemist Analyst*, 1929, *18*, No. 6, p. 5) recommends the addition of 2 grams of chemically pure stick sodium hydroxide to each liter of standard solution of sodium thiosulphate as a stabilizing agent.

J. S. H.

Occurrence of the Pellagra-Preventive Vitamin in Canned Salmon. JOSEPH GOLDBERGER AND G. A. WHEELER (*Public Health Reports*, 1929, *44*, 2769–2771) find that canned salmon contains the pellagra-preventive factor or vitamin P-P, and may be used as a fair substitute for meat in the diet in regions where pellagra is endemic and meat is not readily available.

J. S. H.

The Ratio of the Electromagnetic to the Electrostatic Unit of Electricity as Compared to the Velocity of Light. H. L. CURTIS. (*Bureau of Standards J. of Research*, July, 1929.) No determination of the ratio of the electromagnetic to the electrostatic unit of

electricity has been made since Rosa and Dorsey published their result in 1907. According to Maxwell's Electromagnetic Theory of Light the ratio should be numerically equal to the velocity of light, but their value of the ratio differed from the velocity by 3 parts in 10,000. There have been made since 1907 two determinations that bring about a better agreement. Rosa and Dorsey expressed their result in terms of the international ohm as its value was then known, but in 1913 F. E. Smith got a slightly different value. This is confirmed by an entirely independent determination made in Germany by Gruenheisen and Giebe. When the new value of the ohm in terms of the absolute ohm are introduced into Rosa and Dorsey's calculations the ratio equals 299,790. Meanwhile Michelson has made a fresh measurement of the velocity of light and finds it to be 299,796 km./sec. "The above results show that, within the limits of present measurement, the ratio of the electrical units agrees with the velocity of light. This is an important confirmation of Maxwell's electromagnetic theory of light."

G. F. S.

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